第 10 章作业

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摘　要：.
关键词：词 1，词 2

Homework (Chapter 10)

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Abstract: Abstract.
Keywords: keyword 1, keyword

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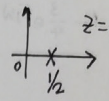
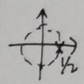
ICE 2301 Homework (Chapter 10) 2022.06.01 (due)

1. 解: (a) $\left(\frac{1}{2}\right)^n u[-n] \leftrightarrow X(z) = \sum_{n=-\infty}^{0} \left(\frac{1}{2z}\right)^n = \sum_{n=0}^{+\infty} (2z)^n = \frac{1}{1-2z}$, $|z| < 1/2$.

$\Rightarrow$ 零点 $z_1 = \infty$, 极点 $p_1 = 1/2$. $\Rightarrow$  , DTFT 不存在.

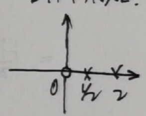
(b) $\left(\frac{1}{2}\right)^n u[n] \leftrightarrow \frac{1}{1-\frac{1}{2}z^{-1}}$, $|z| > 1/2$ $\Rightarrow$ $\left(\frac{1}{2}\right)^{n-1} u[n-1] \leftrightarrow X(z) = \frac{z^{-1}}{1-\frac{1}{2}z^{-1}}$

$= \frac{1}{z-1/2}$, $|z| > 1/2$. $\Rightarrow$ 零点 $z_1 = \infty$, 极点 $p_1 = 1/2$, DTFT 存在. 

(c) $\left(\frac{1}{2}\right)^{|n|} = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1] \leftrightarrow X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}$,

$= \frac{z(z-2) - z(z-1/2)}{(z-\frac{1}{2})(z-2)} = \frac{-\frac{3}{2}z}{(z-1/2)(z-2)}$, $1/2 < |z| < 2$.

DTFT 存在.



3. 解: (a) $X(z) = \frac{10/(1-\frac{1}{2})}{1-\frac{1}{2}z^{-1}} + \frac{10/(1-2)}{1-\frac{1}{4}z^{-1}}$, $|z| > 1/2$

$\leftrightarrow$ $x[n] = \left[20 \cdot \left(\frac{1}{2}\right)^n - 10 \cdot \left(\frac{1}{4}\right)^n\right] u[n]$.

(b) $X(z) = 10 + \frac{10}{(z-1)(z+1)} = 10 + \frac{5}{z-1} + \frac{-5}{z+1}$

$= 10 + \frac{5z^{-1}}{1-z^{-1}} - \frac{5z^{-1}}{1+z^{-1}}$, $|z| > 1$ $\leftrightarrow$ $10\delta[n] + \left[5 \cdot (1)^{n-1} - 5(-1)^{n-1}\right] u[n-1]$.

(c) $\frac{1}{(1-az^{-1})^{m+1}}$, $|z| > |a|$ $\leftrightarrow$ $x[n] = \frac{1}{2\pi j} \oint_{|z|=r} \frac{z^{n-1}}{(1-az^{-1})^{m+1}} dz$, $r > |a|$

$= \frac{1}{2\pi j} \oint_{|z|=r} \frac{z^{m+n}}{(z-a)^{m+1}} dz = \mathrm{Res}\left[\frac{z^{m+n}}{(z-a)^{m+1}}, a\right] u[m+n]$

$= \frac{1}{m!} \frac{d^m}{dz^m}\left(z^{m+n}\right)\Big|_{z=a} = \frac{(m+n)!}{m!\,n!} a^n \underset{u[n+m]}{\cancel{u[n+m]}} = \frac{(n+m)\cdots(n+1)}{m!} a^n u[n]$. − 18 −

$\therefore X(z)=\dfrac{z^{-1}}{(1-6z^{-1})^2}$, $|z|>6 \leftrightarrow x[n]=\dfrac{(n+1)!}{n!}(6)^n u[n]\Big|_{n\leftarrow n-1} = \cancel{n(6)^n u[n]}$

$= (n-1)6^{n-1}u[n-1]$.

3. 解: (d) $\cos(\beta n)u[n] = \frac{1}{2}(e^{j\beta n}+e^{-j\beta n})u[n]$

$\leftrightarrow \frac{1}{2}\left[\dfrac{1}{1-e^{j\beta}z^{-1}}+\dfrac{1}{1-e^{-j\beta}z^{-1}}\right]$, $|z|>1$

$= \dfrac{1-z^{-1}\cos\beta}{1-2\cos\beta\cdot z^{-1}+z^{-2}}$, $|z|>1$,

$\sin(\beta n)u[n] = \frac{1}{2j}(e^{j\beta n}-e^{-j\beta n})u[n]$

$\leftrightarrow \frac{1}{2j}\left[\dfrac{1}{1-e^{j\beta}z^{-1}}-\dfrac{1}{1-e^{-j\beta}z^{-1}}\right]$, $|z|>1$

$= \dfrac{1}{2j}\dfrac{+z^{-1}(e^{j\beta}-e^{-j\beta})}{1-2z^{-1}\cos\beta+z^{-2}} = \dfrac{z^{-1}\sin\beta}{1-2z^{-1}\cos\beta+z^{-2}}$, $|z|>1$.

$\Rightarrow \underbrace{\sin\beta\cos(\beta n)u[n]+\cos\beta\sin(\beta n)u[n]}_{x_0[n]:=} \leftrightarrow \dfrac{\sin\beta}{1-(2\cos\beta)z^{-1}+z^{-2}} =: X_0(z)$, $|z|>1$.

$\Rightarrow X(z)=\dfrac{1+z^{-1}}{\sin\beta}X_0(z) \leftrightarrow x[n]=\dfrac{x_0[n]+x_0[n-1]}{\sin\beta}$ ...

$x_0[n] = \sin[\beta(n+1)]u[n]$, $\Rightarrow x[n]=\dfrac{\sin[\beta(n+1)]+\sin(\beta n)}{\sin\beta}u[n]$.

(e). $X(z)=1-\dfrac{1}{1+z^{-2}}$, $|z|>1 \leftrightarrow \delta[n]-(-1)^n u[n]$.

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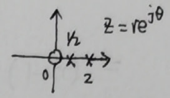
4. 解： $X(z) = \dfrac{-\frac{3}{2}z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})} = \dfrac{-\frac{3}{4}/\frac{3}{4}}{1-2z^{-1}} + \dfrac{-3/(-3)}{1-\frac{1}{2}z^{-1}} = \dfrac{-\frac{3}{2}z}{(z-2)(z-\frac{1}{2})}$.

$\Rightarrow$ 零点：$z_1 = 0$，极点：$p_1 = 2$，$p_2 = 1/2$. $\Rightarrow$ 零-极点图：



(a) $|z| > 2$；右边信号， $x[n] = -(2)^n u[n] + (\frac{1}{2})^n u[n]$.

(b) $|z| < 1/2$；左边信号， $x[n] = 2^n u[-n-1] - (\frac{1}{2})^n u[-n-1]$.

(c) $1/2 < |z| < 2$：双边信号， $x[n] = 2^n u[-n-1] + (\frac{1}{2})^n u[n]$.

5. 解： $x[n-m]u[n] \longleftrightarrow \sum_{n=0}^{+\infty} x[n-m] z^{-(n-m)} z^{m} = z^m\left[\sum_{n=0}^{+\infty} x[n] z^{-n} + \sum_{n=-m}^{-1} x[n] z^{-n}\right]$

$= z^m X(z) + \sum_{n=-m}^{-1} x[n] z^{m-n}$， $m > 0$， 式中 $X(z) := \sum_{n=0}^{+\infty} x[n] z^{-n}$.

(a)

$$Y + \frac{1}{10}\left(z^{-1}Y + y[-1]\right) - \frac{1}{50}\left(z^{-2}Y + z^{-1}y[-1] + y[-2]\right) = \frac{10}{1-z^{-1}}$$

$$\Rightarrow \left(1 + \frac{1}{10}z^{-1} - \frac{1}{50}z^{-2}\right)Y(z) - \left(\frac{1}{50}y[-1]z^{-1} + \frac{1}{50}y[-2] - \frac{1}{10}y[-1]\right) = \frac{10}{1-z^{-1}}$$

$$\Rightarrow Y(z) = \frac{\frac{2}{25}z^{-1} - \frac{7}{25}}{(1+\frac{1}{5}z^{-1})(1-\frac{1}{10}z^{-1})} + \frac{10}{(1-z^{-1})(1+\frac{1}{5}z^{-1})(1-\frac{1}{10}z^{-1})}$$

$$= \frac{\overset{-34/75}{\cancel{-3/25}}}{1+\frac{1}{5}z^{-1}} + \frac{13/75}{1-\frac{1}{10}z^{-1}} + \frac{250/27}{1-z^{-1}} + \frac{-5/3}{1+\frac{1}{5}z^{-1}} + \frac{-10/27}{1-\frac{1}{10}z^{-1}}, \ |z| > 1$$

$$\longleftrightarrow -\left(\frac{34}{75} + \frac{5}{3}\right)\left(-\frac{1}{5}\right)^n + \left(\frac{13}{75} - \frac{10}{27}\right)\left(\frac{1}{10}\right)^n + \frac{250}{27} =: y[n], \quad n \geq 0.$$

5.(b) 解：$\left(1-\frac{9}{10}z^{-1}\right)Y(z)-\frac{9}{10}y[-1]=\frac{1}{20}\frac{1}{1-z^{-1}}$

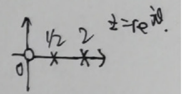
$$\Rightarrow Y(z)=\frac{9/10}{1-\frac{9}{10}z^{-1}}+\frac{1/20}{\left(1-\frac{9}{10}z^{-1}\right)\left(1-z^{-1}\right)}=\frac{\frac{9}{10}-\frac{9}{20}}{1-\frac{9}{10}z^{-1}}+\frac{1/2}{1-z^{-1}},\ |z|>1$$

$$\Leftrightarrow y[n]:=\frac{9}{20}\cdot\left(\frac{9}{10}\right)^n+\frac{1}{2},\quad n\geq 0.$$

6. 解：$\left(z^{-1}-\frac{5}{2}+z\right)Y(z)=X(z)$

(a) $H(z)=\frac{Y(z)}{X(z)}=\frac{z^{-1}}{1-\frac{5}{2}z^{-1}+z^{-2}}=\frac{z^{-1}}{(1-2z^{-1})(1-\frac{1}{2}z^{-1})}=\frac{z}{(z-2)(z-1/2)}$

$\Rightarrow$ 零点：$z_1=0$，极点：$p_1=1/2$，$p_2=2$ $\Rightarrow$ 零-极点图：



(b)(c) $H(z)=\frac{\frac{1}{2}/\frac{3}{4}}{1-2z^{-1}}+\frac{2/(-3)}{1-\frac{1}{2}z^{-1}}$.

(i) ROC：$|z|>2$，因果，非稳定，$h[n]=\frac{2}{3}(2)^n u[n]-\frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$.

(ii) ROC：$1/2<|z|<2$，非因果，稳定，$h[n]=-\frac{2}{3}(2)^n u[-n-1]-\frac{2}{3}\left(\frac{1}{2}\right)^n u[n]$.

(iii) ROC：$0\leq|z|<1/2$，非因果，非稳定，$h[n]=-\frac{2}{3}(2)^n u[-n-1]+\frac{2}{3}\left(\frac{1}{2}\right)^n u[-n-1]$.

7. 解：(a) 当 $x[n]=\left(\frac{1}{2}\right)^n u[n]\Leftrightarrow\frac{1}{1-\frac{1}{2}z^{-1}}$，$|z|>1/2$，时，

$y_{zs}[n]=\delta[n]+a\left(\frac{1}{4}\right)^n u[n]\Leftrightarrow 1+\frac{a}{1-\frac{1}{4}z^{-1}}$，$|z|>1/4$

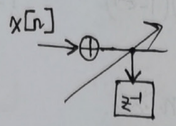
$$\Rightarrow H(z)=\cancel{\frac{\left[(1+a)-\frac{1}{4}z^{-1}\right]}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}}\quad 1-\frac{1}{2}z^{-1}+\frac{a(1-\frac{1}{2}z^{-1})}{1-\frac{1}{4}z^{-1}},\ |z|>1/4.$$

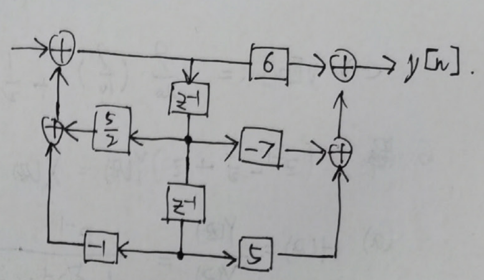
由 $(-2)^n\xrightarrow{H(z)}H(-2)(-2)^n\equiv 0,\forall n\Rightarrow H(-2)=0\Rightarrow a=-9/8$.

(b) $x[n]=(1)^n\xrightarrow{H(z)}y_{zs}[n]=H(1)(1)^n=-\frac{1}{4},\ \forall n.$

8. 解: $H(z)=\dfrac{6-7z^{-1}+5z^{-2}}{1-\frac{5}{2}z^{-1}+z^{-2}}=\dfrac{1}{\Delta}\sum_i g_i\Delta_i$.

⇒ 直接II型:





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References