第 1 次作业

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ICE2301 Chapter 1. Homework 2022.2.24

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1. ~~(a)~~ 节点 $f(t)$: $\{(-1,0), (0,1), (1^-,1), (1^+,2), (2^-,2), (2^+,-1), (3,0)\}$.

(a) $f(t) \xrightarrow{左2} f(t+2) \xrightarrow{\times 1/2} f(2t+2)$. [注：使用上标 $(1^-, 1^+)$ 区分多义点]

设 $y=f(t)$ 上点 $(t_0, \overset{y_0}{\overset{\shortparallel}{f(t_0)}})$ 对应 $y=f(2t+2)$ 上的点 $(t, \overset{y}{\overset{\shortparallel}{f(2t+2)}})$,

$\Rightarrow 2t+2=t_0 \Rightarrow (t_0, y_0) \to (\frac{t_0-2}{2}, y_0)$.

$\therefore$ $y=f(2t+2)$ 节点列: $\{(-\frac{3}{2},0), (-1,1), (-\frac{1}{2}^-,1), (-\frac{1}{2}^+,2), (0^-,2), (0^+,-1), (\frac{1}{2},0)\}$.

(b) $f(t) \overset{左2}{\to} f(t+2) \overset{\times(-3)}{\to} f(-\frac{1}{3}t+2)$.

设 $y=f(t)$ 上的点 $(t_0, y_0=f(t_0))$ 对应 $(t, y=f(2-\frac{t}{3}))$, 则

$\left.\begin{array}{l} 2-\frac{t}{3}=t_0 \\ y=f(2-\frac{t}{3}) \end{array}\right\} \Rightarrow (t_0, y_0) \to (-3(t_0-2), y_0)$.

$\therefore$ $(t, f(2-\frac{t}{3}))$ 节点列: $\{(9,0), (6,1), (3^+,1), (3^-,2), (0^+,2), (0^-,-1), (-3,0)\}$.

(c) $f(t) \overset{左2}{\to} f(t+2) \overset{\times(-1)}{\to} f(-t+2)$: $(t_0, y_0) \to (2-t_0, y_0)$

$\Rightarrow$ $(t, f(-t+2))$ 节点列: $\{\cdots, (1^+,1), (1^-,2), (0^+,2), (0^-,-1), (-1,0)\}$.

$\Rightarrow$ $(t, f(t)+f(2-t))$ 节点列: $\{\cdots, (1^+,3), (1^-,3), (0^+,3), (0^-,0), (-1,0)\}$.

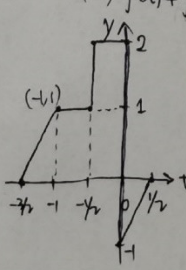

Fig.1(a)

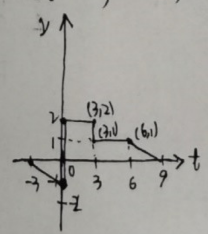

Fig.1(b)

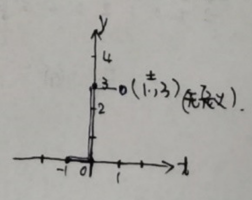

Fig.1(c)

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2. 解：$(n, y=f(n))$ 节点列：

$\{(-5,0), (-4,-1), (-3,-\frac{1}{2}), (-2,\frac{1}{2}), (-1,1), \sim, \sim, (2,1), (3,\frac{1}{2}), (4,0), (5,0)\}$.

(a)

将 $y=f(n)$ 上的点 $(n_0, \overset{y_0=f(n_0)}{\cancel{f(3-n_0)}}) \to y=f(3-n)$ 上的点 $(n, y=f(3-n))$.

$\Rightarrow \begin{cases} 3-n=n_0, \\ y_0=f(n_0) \end{cases} \Rightarrow (n_0, y_0=f(n_0)) \to (3-n_0, y_0)$.

$\therefore$ $(n, y=f(3-n))$ 节点列：

$\{(8,0), (7,-1), (6,-\frac{1}{2}), (5,\frac{1}{2}), (4,1), \sim, \sim, (1,1), (0,\frac{1}{2}), (-1,0), (-2,0)\}$.

(b) $y=f(n)$ 上的点 $(n_0, y_0=f(n_0)) \to y=f(3n+1)$ 上的点 $(n, y=f(3n+1))$.

$\Rightarrow \begin{cases} 3n+1=n_0, \\ y_0=f(n_0) \end{cases} \Rightarrow (n_0, y_0) \to (\frac{n_0-1}{3}, y_0)$.

仅当 $\frac{n_0-1}{3} \in \mathbb{Z}$ 时，上述对应有效.

$\therefore$ $(n, y=f(3n+1))$ 节点列：

$\{(2,1), (1,1), (0,1), (-1,0)\}$.

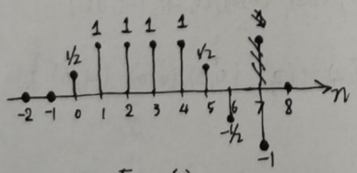

Fig. 2(a)

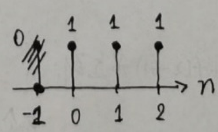

Fig. 2(b)

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3.(a) 解：$f(t)=\frac{1}{2}\left(1+\cos\left(4t-\frac{2}{3}\pi\right)\right) \Rightarrow T=\frac{2\pi}{4}$. (periodic)

(b) 解：$f(t)=\exp\left[j\left(2\pi\cdot\frac{1}{2}t-1\right)\right] \Rightarrow \cancel{N=2}\ T=\frac{2\pi}{\pi}$. (periodic).

4. 解：(a)

$$f(n)=\frac{1}{2}\left[\cos\left(2\pi\cdot\frac{1+\pi}{8\pi}n\right)+\cos\left(2\pi\cdot\frac{1-\pi}{8\pi}n\right)\right]$$

$\Rightarrow$ non-periodic.

(b) $f(n)=\exp\left[j\left(2\pi\cdot\frac{\pi}{16}n-\pi\right)\right] \Rightarrow$ non-periodic.

(c) $f(n)=2\cos\left(2\pi\cdot\frac{1}{8}n\right)+\sin\left(2\pi\cdot\frac{1}{16}n\right)-2\cos\left(2\pi\cdot\frac{1}{4}n+\frac{\pi}{6}\right)$

$\Rightarrow N=\mathrm{LCM}\{8,16,4\}=16.$

(d) ~~设有周期 $N\in\mathbb{N}_+$，则 N 满足~~ $N\in\mathbb{N}_+$ 是周期的充要条件：

$\forall n\in\mathbb{Z},\ \exists k\in\mathbb{Z}:\ \frac{\pi}{8}(n+N)^2=\frac{\pi}{8}n^2+2k\pi$

$\iff N(N-2n)=16k$

$\iff 16\mid N(N-2n)$. $\cdots(*)$

$\xRightarrow{n=0}\ 16\mid N^2 \Rightarrow N=4$ 或 8 或 $\cdots$.

容易验证 $N=4$ 不满足充要条件 $(*)$，而 $N=8$ 满足 $(*)$，故 $N=8$ 为 fundamental period（基波周期）.

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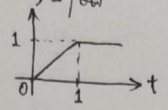
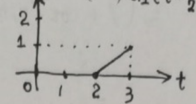
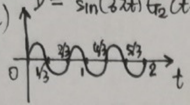
5. 解：(a) $y = f(t) = tG_1(t-\frac{1}{2}) + u(t-1)$



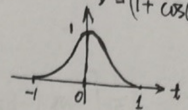
(b) $y = (t-2)G_1(t-\frac{5}{2})$.



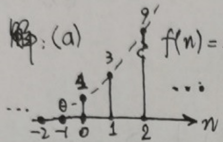
6. 解：(a) $y = \sin(3\pi t)\,G_2(t-1)$



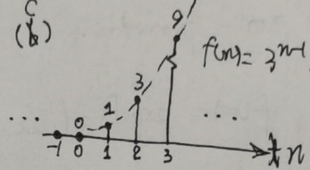
(b) $y = (1+\cos(\pi t))\,G_2(t-0)$



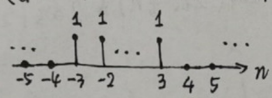
7. 解：(a) $f(n) = 3^n u(n)$



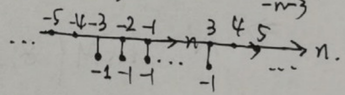
(c) $f(n) = 3^{n-1} u(n-1)$.



8. 解：(a) $f(n) = u(n+3) - u(n-4)$



(b) $f(n) = -\cancel{\delta[n+3]}. \; -\sum_{-n-3}^{-n+3} \delta[m]$.

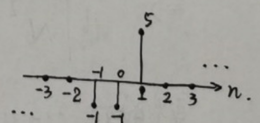


[注] $u[n] = \sum_{m=-\infty}^{n} \delta[m]. \Rightarrow u[n-n_0] = \sum_{m=-\infty}^{n-n_0} \delta[m] \xlongequal{k=m+n_0} \sum_{k=-\infty}^{n} \delta[k-n_0]$.

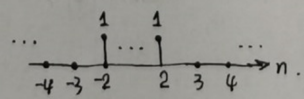
$$\Rightarrow u(-n-4) - u(-n+3) = \left(\sum_{m=-\infty}^{-n-4} - \sum_{m=-\infty}^{-n+3}\right)\delta[m]$$

$$= -\sum_{-n-3}^{-n+3} \delta[m].$$

(c) $f(n) = -\delta(n+1) - \delta(n) + 5\delta(n-1) \Rightarrow$



(d) $f(n) = \sum_{m=-\infty}^{n} \sum_{k=-2}^{2} \delta(m+k) = \sum_{k=-2}^{2} \sum_{m=-\infty}^{n} \delta(m+k) = \sum_{k=-2}^{2} \delta(n+k)$.

$\Rightarrow$ 

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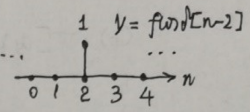
9. 解：(a) $f(t) = \left(1-\frac{1}{2}|t|\right)\left(u(t+2)-u(t-2)\right)$.

(b) $f(t) = u(t)+u(t-1)+u(t-2) = \int_{-\infty}^{t}\sum_{k=0}^{2}\delta(\tau-k)\,d\tau$.

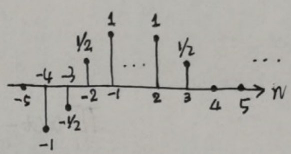
(c) $f(t) = E\sin\left(\frac{\pi}{T}t\right)$.

~~(d)~~

10. 解：(a) $f(n-2)\delta(n-2) = f(0)\delta(n-2)$. $\Rightarrow$



(b) $f(n)u(3-n) = \begin{cases} f(n), & n\le 3,\\ 0, & n\ge 4.\end{cases}$ $\Rightarrow$



11. 解：(a) $f(t) = e^{-1}\delta(t)$.

(b) $\int_{-\infty}^{+\infty}\phi(t)\delta'(t)\,dt = -\phi'(0) \Rightarrow f(t) = -\left(e^{-\tau}\right)'\Big|_{\tau=0} u(t) = u(t)$.

Rmk. $\phi(t)\delta'(t) = [\phi(t)\delta(t)]' - \phi'(t)\delta(t) = \phi(0)\delta'(t) - \phi'(0)\delta(t)$.

(c) $\langle\delta[f(t)],\phi(t)\rangle = \int_{-\infty}^{+\infty}\delta[f(t)]\phi(t)\,dt$

$$= \sum_i \int_{t_i-\varepsilon}^{t_i+\varepsilon}\delta[f(t)]\phi(t)\,dt \xlongequal{\eta=f(t)} \sum_i\int_{t_i-\varepsilon}^{t_i+\varepsilon}\delta(\eta)\frac{\phi(t)}{|f'(t)|}\,d\eta$$

$$= \sum_i \frac{\phi(t_i)}{|f'(t_i)|} = \sum_i\int_{-\infty}^{+\infty}\frac{\delta(t-t_i)}{|f'(t_i)|}\phi(t)\,dt$$

$$= \int_{-\infty}^{+\infty}\phi(t)\sum_i \frac{\delta(t-t_i)}{|f'(t_i)|}\,dt = \left\langle \sum_i \frac{\delta(t-t_i)}{|f'(t_i)|}, \phi(t)\right\rangle.$$

$\Rightarrow \delta[f(t)] = \sum_i \frac{\delta(t-t_i)}{|f'(t_i)|}$. 其中 $t=t_i$ 是 $f(t)$ 的一阶零点.

$\Rightarrow \delta(t^2-4) = \frac{\delta(t+2)}{|2\cdot(-2)|}+\frac{\delta(t-2)}{|2\cdot 2|}$

$\Rightarrow f(t)=\int_{-\infty}^{t} \delta(t^2-4)\,dt = \frac{1}{4}[u(t+2)+u(t-2)]$.

12. 解: (a) $T[a_1x_1(t)+a_2x_2(t)] = \cdots = a_1T[x_1(t)]+a_2T[x_2(t)] \Rightarrow$ Linear.

$T[x(t-t_0)] = x(t-1-t_0) - x(1-t-t_0)$

$\neq x(t-1-t_0) - x(1-t+t_0) = y(t-t_0). \Rightarrow$ not TI.

$y(0) = x(-1) - x(1) \Rightarrow$ not casual.

$|x(t)| \le M \Rightarrow |y(t)| \le 2M. \Rightarrow$ stable.

(b) $T[a_1x_1(t)+a_2x_2(t)] = \begin{cases} 0, & t<0, \\ a_1(x_1(t)+x_1(t-100)) + a_2(x_2(t)+x_2(t-100)), & t\ge 0, \end{cases}$

$= a_1T[x_1(t)] + a_2T[x_2(t)] \Rightarrow$ Linear.

$T[x(t-t_0)] = \begin{cases} 0, & t<0, \\ x(t-t_0)+x(t-(100-t_0)), & t\ge 0 \end{cases}$

$\neq \begin{cases} 0, & t-t_0<0, \\ x(t-t_0)+x(t-t_0-100), & t-t_0\ge 0 \end{cases} = y(t-t_0) \Rightarrow$ not TI.

Obviously casual.

$|x(t)| \le M \Rightarrow |y(t)| \le 2M \Rightarrow$ stable.

(c) $T[a_1x_1(t)+a_2x_2(t)] = \cdots = a_1T[x_1(t)]+a_2T[x_2(t)] \Rightarrow$ Linear.

$T[x(t-t_0)] = x(\frac{t}{2}-t_0) \neq x(\frac{t-t_0}{2}) = y(t-t_0) \Rightarrow$ not TI.

$y(-2) = x(-1) \Rightarrow$ not casual.

$|x(t)| \le M \Rightarrow |y(t)| \le M. \Rightarrow$ stable.

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13. 解: (a) $T[a_1x_1(n)+a_2x_2(n)] = n(a_1x_1(n)+a_2x_2(n)) = a_1T[x_1(n)]+a_2T[x_2(n)]$

$\Rightarrow$ Linear.

$T[x(n-n_0)] = nx(n-n_0) \neq (n-n_0)x(n-n_0) = y(n-n_0)$

$\Rightarrow$ not TI.

Obviously ~~causal~~ causal, not stable.

(b) $T[a_1x_1(n)+a_2x_2(n)] = \sum_{m=n-3}^{n+3}(a_1x_1(m)+a_2x_2(m)) = a_1T[x_1(n)]+a_2T[x_2(n)]$.

$\Rightarrow$ Linear.

$T[x(n-n_0)] = \sum_{m=n-3}^{n+3}x(m-n_0) = \sum_{m=n-n_0-3}^{n-n_0+3}x(m) = y(n-n_0) \Rightarrow$ TI.

Obviously ~~causal~~ causal

$|x(n)| \leq M \Rightarrow |y(n)| \leq 7M \Rightarrow$ stable.

(c) $T[a_1x_1(n)+a_2x_2(n)] = \begin{cases} a_1x_1(n)+a_2x_2(n), & n \geq 1, \\ 0, & n=0, \\ a_1x_1(n+1)+a_2x_2(n+1), & n \leq -1 \end{cases}$

$= a_1T[x_1(n)]+a_2T[x_2(n)]$. $\Rightarrow$ Linear.

$T[x(n-n_0)] = \begin{cases} x(n-n_0), & n \geq 1, \\ 0, & n=0, \\ x(n+1-n_0), & n \leq -1 \end{cases} \neq \begin{cases} x(n-n_0), & n-n_0 \geq 1, \\ 0, & n=n_0, \\ x(n-n_0+1), & n-n_0 \leq -1 \end{cases} = y(n-n_0)$

$\Rightarrow$ not TI.

Obviously causal, stable.

14. 解: (a) invertible. The inverse system is $T[x(t)] = x(t+4)$.

(b) not invertible. $x_1(t) = x(t)$, $x_2(t) = x(t)+2\pi$ 同输出.

(c) not invertible. $x_1(n) = x(n)$, $x_2(n) = -x(n)$ 同输出.

(d) invertible. The inverse system is $T[x(n)] = x(1-n)$.

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