第 3 次作业

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ICE2301 Homework (Ch3) 2022.3.13 (due date)

1\. 解：(a) $y[-1]=0 \Rightarrow y[0]=0^2+y[-1]=0 \Rightarrow y[1]=1^2+y[0]=1$

$\Rightarrow y[2]=2^2+y[1]=5 \Rightarrow$ ~~猜想~~: $y[n]=n^2+y[n-1]=\cdots$

$=n^2+(n-1)^2+y[n-2]=\cdots=n^2+(n-1)^2+\cdots+1^2+y[0]$

$=\dfrac{n(n+1)(2n+1)}{6}, \quad n\geq 0.$

(b) 特征方程 $\alpha-1=0 \Rightarrow \alpha=1 \Rightarrow y_h=C\cdot 1^n=C.$

$\Rightarrow y_p=(B_1n^2+B_2n+B_3)n \xRightarrow{\text{代入方程}} n^2(3B_1)+n(-3B_1+2B_2)$

$+(B_1-B_2+B_3)=n^2 \Rightarrow B_1=1/3,\ B_2=1/2,\ B_3=1/6$

$\Rightarrow y=y_h+y_p=\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n+C \xRightarrow{y[-1]=0} C=0$

$\Rightarrow y[n]=\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n+0=\dfrac{n(n+1)(2n+1)}{6}, \quad n\in\mathbb{Z}.$

2\. (1) 解：特征方程 $\alpha^2+3\alpha+2=0 \Rightarrow \alpha_1=-1,\ \alpha_2=-2$

$$\left.\begin{array}{l} \Rightarrow y_{zi}=C_{zi_1}(-1)^n+C_{zi_2}(-2)^n \\ y_{zi}[-1]=1,\ y_{zi}[-2]=0 \end{array}\right\} \Rightarrow y_{zi}=(-1)^n-4(-2)^n, \quad n\in\mathbb{Z}.$$

设 $y_{zs}=[C_{zs_1}(-1)^n+C_{zs_2}(-2)^n+y_p]u[n]$，其中 $y_p=B_0 \xRightarrow{\text{代入方程}} y_p=1/6$ ~~u[n]~~

$$\left.\begin{array}{l} \Rightarrow y_{zs}=\left[C_{zs_1}(-1)^n+C_{zs_2}(-2)^n+\frac{1}{6}\right]u[n], \\ y_{zs}[n]=0\ (\forall n<0) \Rightarrow y_{zs}[0]=1 \Rightarrow y_{zs}[1]=-2 \end{array}\right\} \Rightarrow C_{zs_1}=-1/2,\ C_{zs_2}=4/3$$

$$\Rightarrow y=y_{zi}+y_{zs}=\begin{cases} \frac{1}{2}(-1)^n-\frac{8}{3}(-2)^n+1/6, & n\geq 0, \\ (-1)^n-4(-2)^n, & n<0. \end{cases}$$

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2.(2) 解：特征方程 $\alpha^2-\alpha-2=0 \Rightarrow \alpha_1=-1,\ \alpha_2=2$

$$\left.\begin{aligned} \Rightarrow y_{zi} &= C_{zi_1}(-1)^n + C_{zi_2}\cdot 2^n,\quad n\in\mathbb{Z}. \\ y_{zi}[-1] &= 2,\quad y_{zi}[-2] = -1/2 \end{aligned}\right\} \Rightarrow C_{zi_1}=-1,\ C_{zi_2}=2. \qquad (1)$$

设 ~~第~~ ~~y_1~~ $\begin{cases} y[n]-y[n-1]-2y[n-2] = x[n]\ (=u[n]) \\ y[n]=0,\ \forall n<0 \Rightarrow y[0]=1,\ y[1]=2 \end{cases} \cdots (*)$

的解为

$$y_1 = \left[C_{zs_1}^{(1)}\cdot(-1)^n + C_{zs_2}^{(1)}\cdot 2^n + y_p^{(1)}\right]u[n],$$

其中 $y_p^{(1)} = B^{(1)} \xRightarrow{代入(*)} \begin{cases} y_1 = \left[C_{zs_1}^{(1)}(-1)^n + C_{zs_2}^{(1)}\cdot 2^n - \frac{1}{2}\right]u[n], \\ y_1[0]=1,\ y_1[1]=2 \end{cases}$

$$\Rightarrow y_1 = \left[\frac{1}{6}(-1)^n + \frac{4}{3}\cdot 2^n - \frac{1}{2}\right]u[n].$$

~~(2)~~

$\xRightarrow{LSI} \begin{cases} y[n]-y[n-1]-2y[n-2] = 2x[n-2], \\ y[n]=0,\ \forall n<0 \end{cases}$ 的解为

$$y_2 = 2y_1[n-2]$$

$$\Rightarrow y_{zs} = y_1 + y_2 = y_1[n] + 2y_1[n-2]. \qquad (2)$$

$\Rightarrow y = y_{zi} + y_{zs}$，其中 y_{zi}，y_{zs} 是 (1)(2).

3. 解：

$$\left.\begin{aligned} x[n] &\xrightarrow{y(0)} y_1 = y_{zi} + y_{zs}, \\ -x[n] &\xrightarrow{y(0)} y_2 = y_{zi} - y_{zs} \end{aligned}\right\} \Rightarrow y_{zi} = \frac{y_1+y_2}{2},\quad y_{zs} = \frac{y_1-y_2}{2}$$

$$\Rightarrow 4x[n] \xrightarrow{2y(0)} 2y_{zi} + 4y_{zs} = 3y_1 - y_2,$$

其中 $y_1 = [1+(\frac{1}{2})^n]u[n]$，$y_2 = [(-\frac{1}{2})^n - 1]u[n]$.

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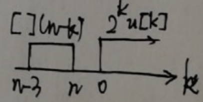
4. 解：(1) $2^n u[n] * 3^n u[n] = \left[\sum_{k=0}^{n} 2^k u[k]\, 3^{n-k} u[n-k] \right] u[n]$

$$= 3^n u[n] \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k = 3^n u[n] \frac{1-\left(\frac{2}{3}\right)^{n+1}}{1-2/3}.$$

(2) $2^{-n} u[-n] * 3^{-n} u[-n] = u[0-n] \sum_{k=n}^{0} 2^{-k} u[-k]\, 3^{k-n} u[k-n]$

$$= u[-n]\, 3^{-n} \sum_{k=n}^{0} \left(\frac{3}{2}\right)^k = u[-n] 3^{-n} \left(\frac{3}{2}\right)^n \frac{1-\left(\frac{3}{2}\right)^{1-n}}{1-3/2}.$$

5. 解：(1)



1°. $n<0$: $y=0$.

2°. $0 \le n \le 3$: $y = \sum_{k=0}^{n} 2^k = \frac{1-2^{n+1}}{1-2}$,

3°. $n>3$: $y = \sum_{k=n-3}^{n} 2^k = 2^{n-3} \frac{1-2^4}{1-2}$.

$$\Rightarrow 2^n u[n] * [u[n]-u[n-4]] = \left[\begin{cases} 2^{n+1}-1, & 0 \le n \le 3, \\ 15 \cdot 2^{n-3}, & n>3, \\ 0, & \text{otherwise}. \end{cases} \right]$$

(2) $\cos\frac{n\pi}{2} * \left[\sin\frac{n\pi}{2}[u[n]-u[n-5]]\right] *$

$$= \cos\frac{n\pi}{2} * \left[\sin\frac{n\pi}{2} \cdot (\delta[n] + \delta[n-1] + \cdots + \delta[n-4])\right]$$

$$= \cos\frac{n\pi}{2} * [\delta[n-1] - \delta[n-3]] = \cos\frac{(n-1)\pi}{2} - \cos\frac{(n-3)\pi}{2}$$

$$= 2\sin\frac{n\pi}{2} = \begin{cases} 0, & n=2k, \\ 2\cdot(-1)^{k+1}, & n=2k+1, \end{cases} \quad k \in \mathbb{Z}.$$

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6. 解：LTI 系统:因果 $\Longleftrightarrow h[n]=0\ (\forall n<0)$,

LTI 系统:稳定 $\Longleftrightarrow \sum_{n=-\infty}^{+\infty}|h[n]| < +\infty$.

(a) ~~h[-1] ≠ 0~~ ⇒ 显然非因果； $\sum_n |h[n]| = \sum_{n=0}^{+\infty}\left(\frac{1}{2}\right)^n = \frac{1}{1-1/2} < \infty$ ⇒ 稳定；

(b) ~~h[n] = 0 (∀n~~ 非因果； $\sum_n |h[n]| = \sum_{n=-\infty}^{0}\left(\frac{1}{2}\right)^n = \sum_{n=0}^{+\infty} 2^n \to +\infty$
⇒ 不稳定.

(c) $h[-1] \neq 0$ ⇒ 非因果； $\sum_n |h[n]| = \sum_{n=-\infty}^{2} 3^n = \sum_{n=-2}^{+\infty}\left(\frac{1}{3}\right)^n < \infty$ ⇒ 稳定；

(d) $h[n]=0\ (\forall n<0)$ ⇒ 因果； $\sum_n |h[n]| = \sum_{n=0}^{+\infty} 3 \to +\infty$ ⇒ 不稳定,.

7. 解：$h[n] = h_1[n] * (h_2[n] - h_3[n])$

$= u[n] * (\delta[n] - \delta[n-N]) = u[n] - u[n-N]$.

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