第 4 章作业

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摘　要：求助：Chapter 4-3 Q5(2) 寻求物理解释？.

关键词：词 1，词 2

Homework (Chapter 4)

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Abstract: Abstract.

Keywords: keyword 1, keyword

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2022 Spring ICE2301 Homework (Chapter 4-1) Q2

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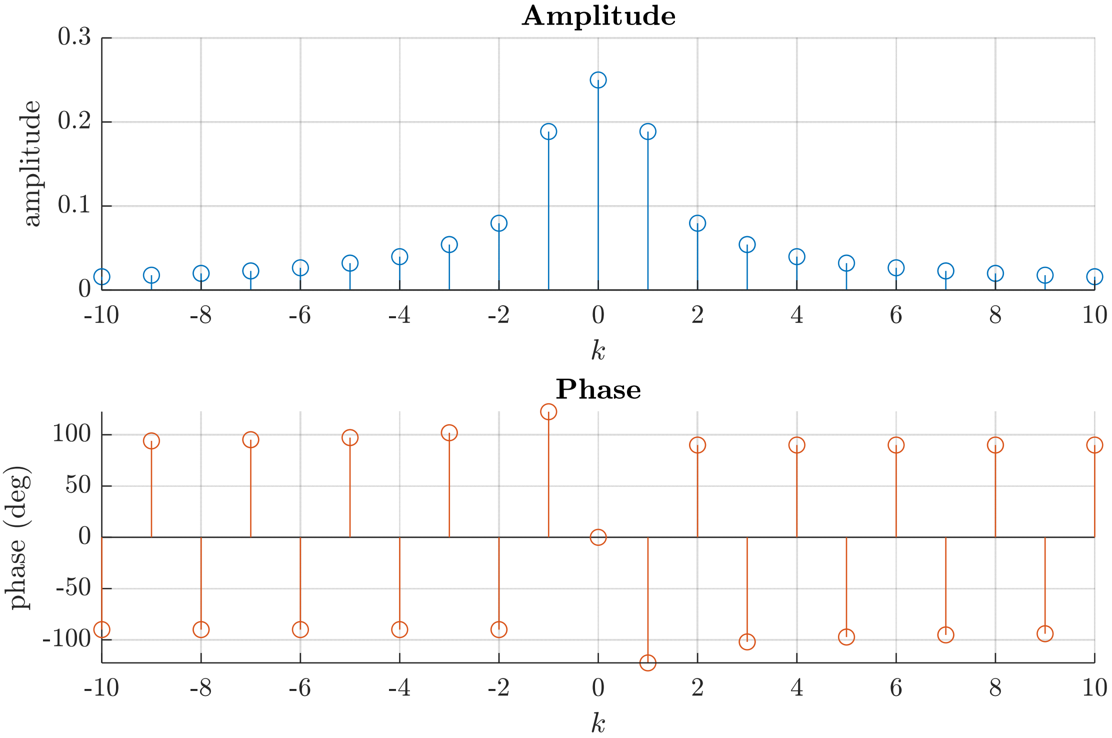

图 1　第 4-1 章作业第 2 题图

2022 Spring ICE2301 Homework (Chapter 4-1) Q5(a)

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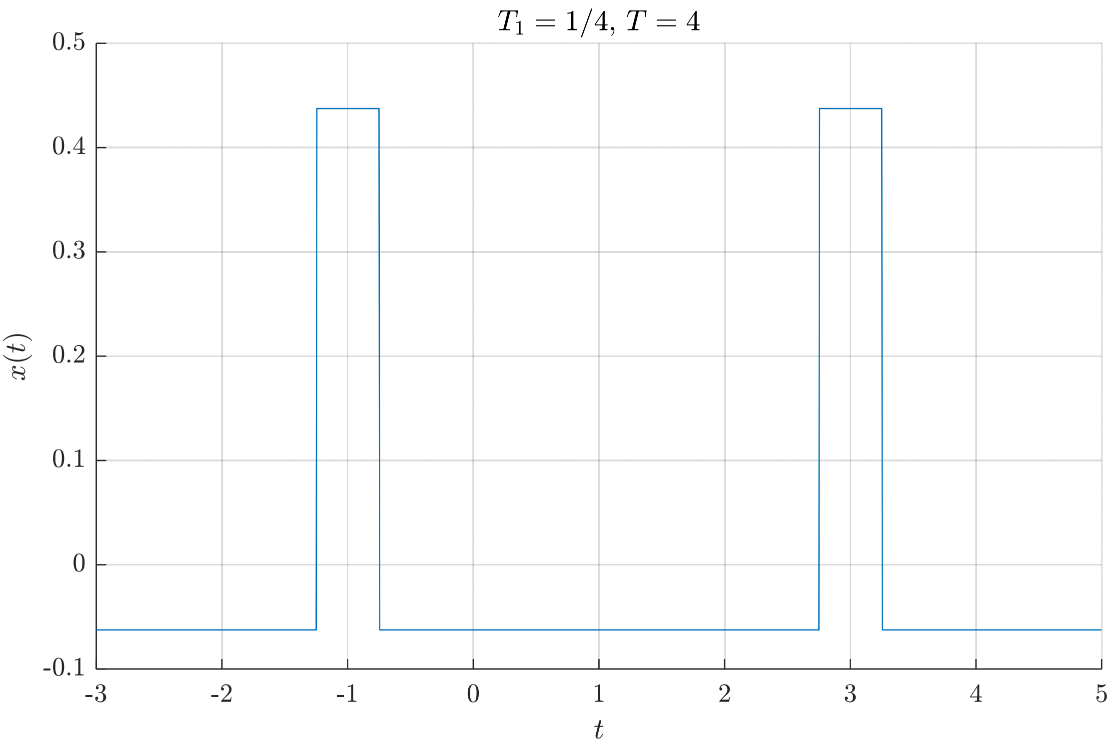

图 2　第 4-1 章作业第 5(a)题图

2022 Spring ICE2301 Homework (Chapter 4-3) Q5(a)

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$$h(t) = \frac{2}{\pi}\cos(\omega_0 t)\frac{\sin(\omega_c(t-t_0))}{t-t_0}, \quad \omega_c = 1,\ \omega_0 = 2,\ t_0 = 1$$

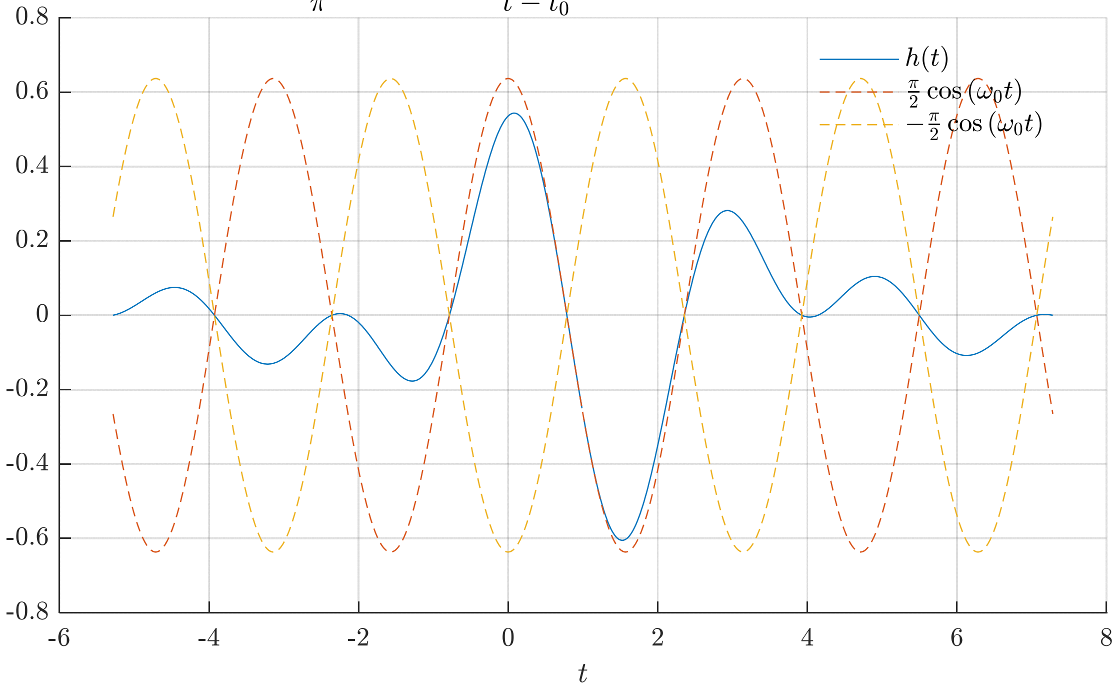

图 3　第 4-3 章作业第 5(a)题图

2022 Spring ICE2301 Homework (Chapter 4-3) Q12(a)

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$$H(\omega) = \frac{1/2}{j\omega+1} + \frac{3/2}{j\omega+3}$$

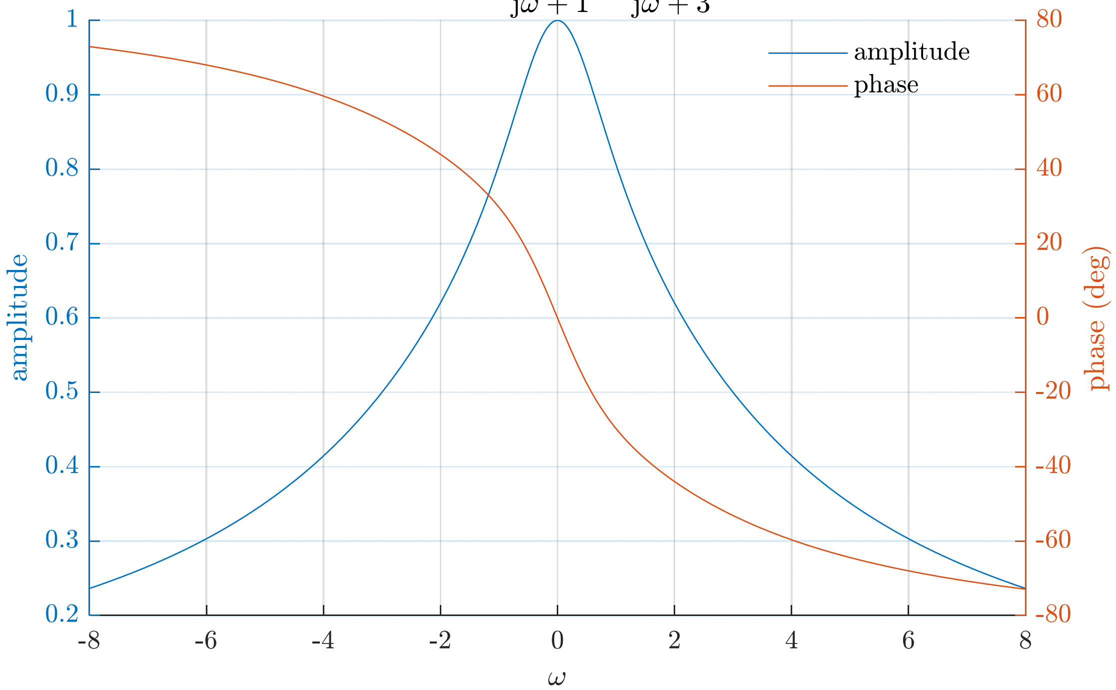

图 4　第 4-3 章作业第 12(a)题图

1 Chapter 4-1

ICE2301 Homework (chapter 4-1) 2022.04.17
04.24
(due date)

1. 解: (1) 指数形式: $f(t)=\sum_k a_k e^{jk\omega_0 t}$, $\omega_0=\frac{2\pi}{T}$.

$$\Rightarrow a_k = \frac{\langle f(t), e^{jk\omega_0 t}\rangle}{\langle e^{jk\omega_0 t}, e^{jk\omega_0 t}\rangle} = \frac{1}{T}\int_T \underbrace{f(t)e^{-jk\omega_0 t}}_{奇} dt$$

$$= \frac{1}{T}\cdot 2\cdot\int_0^{T/2}\frac{E}{2}\cdot -j\sin(k\omega_0 t)\,dt = \frac{jE}{2\pi k}\left[(-1)^k-1\right] = \begin{cases}0, & k=2n,\\ -\frac{jE}{\pi k}, & k=2n+1.\end{cases} \quad n\in\mathbb{Z}.$$

$$\Rightarrow f(t)=\sum_n -\frac{jE}{(2n+1)\pi}e^{j(2n+1)\omega_0 t}, \quad \omega_0=\frac{2\pi}{T}.$$

(2) 三角形式: $f(t)=\sum_{n=0}^{+\infty} a_n\cos(n\omega_0 t)+b_n\sin(n\omega_0 t)$, $\omega_0=\frac{2\pi}{T}$.

$$\Rightarrow a_n = \frac{\langle f(t), \cos(n\omega_0 t)\rangle}{\langle \cos(n\omega_0 t), \cos(n\omega_0 t)\rangle} = \frac{\omega_0}{\pi}\int_{-T/2}^{T/2}\underbrace{f(t)\cos(n\omega_0 t)}_{奇}dt = 0,$$

$$b_n = \frac{\omega_0}{\pi}\int_{-T/2}^{T/2} f(t)\sin(n\omega_0 t)\,dt = \frac{\omega_0}{\pi}\cdot 2\cdot\int_0^{T/2}\frac{E}{2}\sin(n\omega_0 t)\,dt$$

$$= \frac{\cancel{\omega_0} E}{n\pi\cancel{\omega_0}}\cdot\left[-\cos(n\omega_0 t)\right]_{t=0}^{T/2} = \frac{E}{n\pi}\left[1-(-1)^n\right] = \begin{cases}0, & n=2k+2,\\ \frac{2E}{n\pi}, & n=2k+1,\end{cases} \quad k\in\mathbb{N}$$

$$\Rightarrow f(t)=\sum_{k=0}^{+\infty} \cancel{\frac{2E}{n\pi}}\,\frac{2E}{(2k+1)\pi}\sin\left[(2k+1)\omega_0 t\right], \quad \omega_0=\frac{2\pi}{T}.$$

2. 解: (1) 指数形式: $f(t)=\sum_k a_k e^{jk\omega_0 t}$, $\omega_0=\frac{2\pi}{T}$.

$$a_k = \frac{1}{T}\int_0^{T/2}\frac{t}{T/2}e^{-jk\omega_0 t}\,dt = \frac{(-1)^{k+1}}{j2k\pi}+\frac{(-1)^k-1}{2(k\pi)^2}, \quad k\in\mathbb{Z}^*$$

$$= \begin{cases}\frac{j}{2k\pi}, & k=2n,\\ \frac{-j}{2k\pi}+\frac{-1}{\pi^2k^2}, & k=2n+1,\end{cases} \quad n\in\mathbb{Z}^*; \quad a_0=\frac{1}{T}\int_0^{T/2}\frac{t}{T/2}\,dt=\frac{1}{4}.$$

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$$\Rightarrow |a_k| = \begin{cases} \cancel{(2k\pi)^{-1}} \frac{1}{2|k|\pi}, & k=2n, \\ \left[(2k\pi)^{-2} + (k\pi)^{-4}\right]^{1/2}, & k = 2n+1, \end{cases} \quad n \in \mathbb{Z}^*,$$

$$|a_0| = 1/4, \quad \arg a_0 = 0.$$

$$\arg a_k = \begin{cases} -\arctan\frac{k\pi}{2}, & k=2n+1 \ \cancel{k\in\mathbb{Z}}, \\ \frac{\pi}{2}\frac{k}{|k|}, & k=2n, \end{cases} \quad n\in\mathbb{Z}^*,$$

(2) 三角形式. $f(t) = a_0 + \sum_{n=1}^{+\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$, $\omega_0 = \frac{2\pi}{T}$.

$$\Rightarrow a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt = \frac{1}{T}\int_0^{T/2} \frac{t}{T/2}\,dt = \frac{1}{4},$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega_0 t)\,dt = \frac{2}{T}\int_0^{T/2}\frac{t}{T/2}\cos(n\omega_0 t)\,dt$$

$$= \frac{4}{T^2}\left[\frac{t}{n\omega_0}\sin(n\omega_0 t) + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\right]_{t=0}^{T/2} = \frac{(-1)^n - 1}{(n\pi)^2}, \quad n \geq 1,$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega_0 t)\,dt = \frac{2}{T}\int_0^{T/2}\frac{t}{T/2}\sin(n\omega_0 t)\,dt$$

$$= \frac{4}{T^2}\left[-\frac{t}{n\omega_0}\cos(n\omega_0 t) + \frac{\sin(n\omega_0 t)}{(n\omega_0)^2}\right]_{t=0}^{T/2} = \frac{(-1)^{n+1}}{n\pi}, \quad n\geq 1.$$

3. 解: (2). $f(t) = \begin{cases} \frac{t}{T/2}\left[u(t) - u(t-\frac{T}{2})\right], & -T/2 < t \leq T/2, \\ f(t+T), & \forall t \in \mathbb{R}. \end{cases}$

$$\Rightarrow f'(t) = \begin{cases} -\delta(t-\frac{T}{2}) + \frac{2}{T}\left[u(t) - u(t-\frac{T}{2})\right], & -\frac{T}{2} < t \leq \frac{T}{2}, \\ f'(t+T), & \forall t\in\mathbb{R}. \end{cases}$$

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设 $f(t)=\sum_k a_k e^{jk\omega_0 t}$，则 $f'(t)=\sum_k (jk\omega_0 a_k)e^{jk\omega_0 t}$，$\omega_0=\frac{2\pi}{T}$.

$$\Rightarrow jk\omega_0 a_k = \frac{1}{T}\int_{-T/2}^{T/2} f'(t)e^{-jk\omega_0 t}dt$$

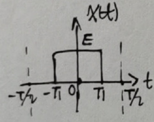
$$= \frac{1}{T}\left[-e^{-jk\omega_0\frac{T}{2}} + \frac{2}{T}\int_0^{T/2} e^{-jk\omega_0 t}dt\right]$$

$$= \frac{1}{T}\left[-(-1)^k + \frac{2}{T}\frac{(-1)^k-1}{-jk\omega_0}\right]$$

$$\Rightarrow \begin{cases} a_k = \dfrac{j(-1)^k}{2k\pi} + \dfrac{(-1)^k-1}{2(k\pi)^2}, & k\neq 0. \\ a_0 = \dfrac{1}{T}\displaystyle\int_{-T/2}^{T/2} f(t)\,dt = \dfrac{1}{T}\int_0^{T/2}\dfrac{t}{T/2}dt = \dfrac{2}{T^2}\dfrac{1}{2}\dfrac{T^2}{4} = \dfrac{1}{4}. \end{cases}$$

这与第2题的结果相同.

3(1) 解：



$$g(t;T_1,T,E) := \begin{cases} E, & |t|<T_1<T/2, \\ 0, & T_1<|t|<T/2, \\ x(t+T), & \forall t\in\mathbb{R}. \end{cases}$$

$$\Rightarrow g(t;T_1,T,E) = \sum_k \hat{g}_k e^{jk\omega_0 t},\quad \omega_0=\frac{2\pi}{T},\quad \hat{g}_k = \hat{g}_k(T_1,T,E).$$

$$\hat{g}_k = \frac{E}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}dt = \left.\frac{Ee^{-jk\omega_0 t}}{-jk\omega_0 T}\right|_{-T_1}^{T_1} = \frac{-E\,2j\sin(k\omega_0 T_1)}{-jk\,2\pi}$$

$$= \frac{E\sin(k\omega_0 T_1)}{k\pi},\quad k\neq 0.$$

$$\hat{g}_0 = \frac{E}{T}\int_{-T_1}^{T_1} 1\,dt = \frac{2ET_1}{T}.$$

观察出 $f(t+\frac{T}{4})+\frac{E}{2} = g(t;\frac{T}{4},T,E) \Rightarrow f(t) = -\frac{E}{2}+g(t-\frac{T}{4};\frac{T}{4},T,E)$.

因 $-\frac{E}{2} \leftrightarrow a_k := \begin{cases}-E/2, & k=0, \\ 0, & k\neq 0,\end{cases}$ $g(t-\frac{T}{4};\frac{T}{4},T,E) \leftrightarrow b_k = e^{-jk\omega_0\frac{T}{4}}\hat{g}_k(\frac{T}{4},T,E)$

$= (-j)^k\cdot\frac{E\sin(\frac{k}{2}\pi)}{k\pi}$，$k\neq 0$，$b_0=\frac{E}{2}$.

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$$\Rightarrow f(t) = \sum_k \hat{f}_k e^{jk\omega_0 t}, \quad \hat{f}_k = a_k + b_k = \begin{cases} 0, & k=0, \\ (-j)^k \frac{E}{k\pi}\sin(\frac{k}{2}\pi), & k\neq 0. \end{cases}$$

$$\Rightarrow \hat{f}_k = \begin{cases} 0, & \cancel{n=0}, k=2n, \\ -\frac{jE}{(2n+1)\pi}, & \cancel{n\neq 0}\ k=2n+1, \end{cases} \quad n\in\mathbb{Z}.$$

$$\Rightarrow f(t) = \sum_n -\frac{jE}{(2n+1)\pi} e^{jk\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}.$$

4. 解：

$$g(t;T_1,T,E) := \begin{cases} E, & |t|<T_1<T/2, \\ 0, & T_1<|t|<T/2, \\ g(t+T;T_1,T,E), & \forall t\in\mathbb{R}. \end{cases}$$

$$\Rightarrow g(t;T_1,T,E) = \sum_k \hat{g}_k(T_1,T,E) e^{jk\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}, \quad \hat{g}_k = \begin{cases} \frac{E\sin(k\omega_0 T_1)}{k\pi}, & k\neq 0, \\ 2ET_1/T, & k=0. \end{cases}$$

$\therefore$ 圆频率分量 $\omega = 2\pi f$ 不为零，当且仅当：

$\cancel{\exists n\in\mathbb{Z}}$ "$f/f_0 \in \mathbb{Z}$ 且 $2fT_1 \notin \mathbb{Z}$". (1)

当 $f_0 = 5\times 10^3$ Hz, $T_1 = \tau/2 = 10^{-5}$ s 时，

当 $f = 5, 20, 80$ kHz 时，条件(1) 满足，相应频率分量非零，

当 $f = 12, 50, 100$ kHz 时，条件(1) 不成立，相应频率分量为零。

5.(b) 解：

$$x(t) = a_0 + \sum_{k=1}^{+\infty} a_k e^{jk\omega_0 t} + a_{-k} e^{-jk\omega_0 t} \overset{a_{-k}=-a_k}{=\!=\!=} \sum_{k=1}^{+\infty} a_k \left(e^{jk\omega_0 t} - e^{-jk\omega_0 t}\right)$$

$$= \sum_{k=1}^{+\infty} a_k \cdot 2j\sin(k\omega_0 t) \xlongequal[a_k=0\ (\text{otherwise})]{a_{\pm1}=\pm j,\ a_{\pm2}=\pm 2j} -2\sin(\omega_0 t) - 4\sin(2\omega_0 t),$$

$$\omega_0 = \frac{2\pi}{T} = \pi/2.$$

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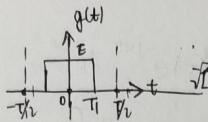
5 (a) 解:



记 $g(t;T_1,T,E) := \begin{cases} E, & |t| < T_1 < T/2, \\ 0, & T_1 < |t| < T/2, \\ g(t+T;T_1,T,E), & \forall t \in \mathbb{R}. \end{cases}$

$$\Rightarrow g(t;T_1,T,E) = \sum_k \hat{g}_k(T_1,T,E)e^{jk\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T},$$

$$\hat{g}_k = \hat{g}_k(T_1,T,E) = \frac{1}{T}\int_{-T_1}^{T_1} E e^{-jk\omega_0 t}\,dt = \begin{cases} \dfrac{E\sin(k\omega_0 T_1)}{k\pi} = \dfrac{E}{k\pi}\sin\left(\dfrac{2T_1}{T}k\pi\right), & k \neq 0, \\ \dfrac{E\omega_0 T_1}{\pi} = \dfrac{2ET_1}{T}, & k = 0. \end{cases}$$

$$\Rightarrow \hat{g}_k\left(\frac{1}{4}, 4, \overset{1}{\cancel{E}}\right) = \begin{cases} \sin\left(\frac{k}{8}\pi\right)/k\pi, & k \neq 0, \\ E/8, & k = 0, \end{cases} \quad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\Rightarrow a_k = \begin{cases} 0, & k = 0, \\ e^{jk\frac{\pi}{2}}\dfrac{\sin(k\pi/8)}{k\pi}, & k \neq 0 \end{cases} = \frac{1}{2}e^{jk\frac{\pi}{2}}\hat{g}_k + \begin{cases} -E/16, & k = 0, \\ 0, & k \neq 0. \end{cases}$$

因 $\frac{1}{2}e^{jk\frac{\pi}{2}}\hat{g}_k \Longleftrightarrow \frac{1}{2}g(t+\overset{1}{\cancel{\tfrac{\pi}{2}}})$, $\begin{cases} -E/16, & k = 0, \\ 0, & k \neq 0 \end{cases} \Longleftrightarrow -\frac{E}{16}$.

故 $a_k \leftrightarrow \overset{\cancel{f}x(t)=}{\sum_k a_k e^{jk\omega_0 t}} = \frac{1}{2}g(t+\overset{1}{\cancel{\tfrac{\pi}{2}}}, \frac{1}{4}, 4, 1) - \frac{E}{16}$, $E = 1$.

chapter 4-2.

1. 解: (1) $\mathcal{F}[f(t)] := \int_{\mathbb{R}} f(t)e^{-j\omega t}\,dt = \int_{-T/2}^{T/2}\frac{2E}{T}t\,e^{-j\omega t}\,dt = -j\frac{4E}{T}\int_0^{T/2} t\sin(\omega t)\,dt$

$$= \begin{cases} -j\dfrac{4E}{T}\left[-\dfrac{T}{2\omega}\cos\left(\omega\dfrac{T}{2}\right) + \dfrac{1}{\omega^2}\sin\left(\omega\dfrac{T}{2}\right)\right], & \omega \neq 0, \\ 0, & \omega = 0. \end{cases}$$

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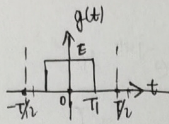
2 Chapter 4-2

5 (a) 解:  记 $g(t;T_1,T,E) := \begin{cases} E, & |t| < T_1 < T/2, \\ 0, & T_1 < |t| < T/2, \\ g(t+T;T_1,T,E), & \forall t \in \mathbb{R}. \end{cases}$

$$\Rightarrow g(t;T_1,T,E) = \sum_k \hat{g}_k(T_1,T,E)e^{jk\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T},$$

$$\hat{g}_k = \hat{g}_k(T_1,T,E) = \frac{1}{T}\int_{-T_1}^{T_1} E e^{-jk\omega_0 t}\,dt = \begin{cases} \frac{E\sin(k\omega_0 T_1)}{k\pi} = \frac{E}{k\pi}\sin\left(\frac{2T_1}{T}k\pi\right), & k \neq 0, \\ \frac{E\omega_0 T_1}{\pi} = \frac{2ET_1}{T}, & k = 0. \end{cases}$$

$$\Rightarrow \hat{g}_k\left(\frac{1}{4}, 4, \overset{1}{\cancel{E}}\right) = \begin{cases} \sin\left(\frac{k}{8}\pi\right)/k\pi, & k \neq 0, \\ E/8, & k = 0, \end{cases} \quad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\Rightarrow a_k = \begin{cases} 0, & k = 0, \\ e^{jk\frac{\pi}{2}}\frac{\sin(k\pi/8)}{k\pi}, & k \neq 0 \end{cases} = \frac{1}{2}e^{jk\frac{\pi}{2}}\hat{g}_k + \begin{cases} -E/16, & k = 0, \\ 0, & k \neq 0. \end{cases}$$

因 $\frac{1}{2}e^{jk\frac{\pi}{2}}\hat{g}_k \Leftrightarrow \frac{1}{2}g\left(t+\overset{1}{\cancel{\frac{\pi}{2}}}\right)$, $\begin{cases} -E/16, & k = 0, \\ 0, & k \neq 0 \end{cases} \longleftrightarrow -\frac{E}{16}$.

故 $a_k \leftrightarrow \overset{f(t)=}{\sum_k a_k e^{jk\omega_0 t}} = \frac{1}{2}g\left(t+\overset{1}{\cancel{\frac{\pi}{2}}}, \frac{1}{4}, 4, 1\right) - \frac{E}{16}$, $E = 1$.

chapter 4-2.

1. 解: (1) $\mathcal{F}[f(t)] := \int_{\mathbb{R}} f(t)e^{-j\omega t}\,dt = \int_{-T/2}^{T/2}\frac{2E}{T}t\,e^{-j\omega t}\,dt = -j\frac{4E}{T}\int_0^{T/2} t\sin(\omega t)\,dt$

$$= \begin{cases} -j\frac{4E}{T}\left[-\frac{T}{2\omega}\cos\left(\omega\frac{T}{2}\right) + \frac{1}{\omega^2}\sin\left(\omega\frac{T}{2}\right)\right], & \omega \neq 0, \\ 0, & \omega = 0. \end{cases}$$

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1.(2) 解: $\mathcal{F}[f(t)] := \int_{\mathbb{R}} f(t)e^{-j\omega t}dt = \int_0^T E\sin(\frac{2\pi}{T}t)e^{-j\omega t}dt$

$$= E\cdot\left[\begin{array}{c|c|c} \sin\frac{2\pi}{T}t & \frac{2\pi}{T}\cos(\frac{2\pi}{T}t) & -\frac{4\pi^2}{T^2}\sin(\frac{2\pi}{T}t) \\ e^{-j\omega t} \quad \oplus & \frac{1}{-j\omega}e^{-j\omega t} \quad \ominus & -\frac{1}{\omega^2}e^{-j\omega t} \quad \oplus \end{array}\right]_{t=0}^{T}$$

$$=\begin{cases} E\cdot\dfrac{1}{1-\frac{4\pi^2}{\omega^2T^2}}\cdot\dfrac{2\pi}{\omega^2T}\cdot\left(e^{-j\omega T}-1\right) = \dfrac{2\pi ET}{\omega^2T^2-4\pi^2}\left(e^{-j\omega T}-1\right), & \omega\neq 0. \\ 0, & \omega=0. \end{cases}$$

2(3) 解: (1) $e^{-t}u(t) \leftrightarrow \int_0^{+\infty} e^{-(1+j\omega)t}dt = \left.\frac{e^{-(1+j\omega)t}}{-(1+j\omega)}\right|_0^{+\infty} = \frac{1}{1+j\omega}$.

[$x(t) = \frac{1}{2\pi}\int_{\mathbb{R}} X(\omega)e^{j\omega t}d\omega \Rightarrow x(-t) = \frac{1}{2\pi}\int_{\mathbb{R}} X(\omega)e^{-j\omega t}d\omega = \frac{1}{2\pi}\int_{\mathbb{R}} X(-\omega)e^{j\omega t}d\omega$

$\Rightarrow \therefore\ x(-t) \leftrightarrow X(-\omega)$]

$\Rightarrow e^{t}u(-t) \leftrightarrow \frac{1}{1-j\omega}$. $\quad 1 \leftrightarrow \int_{\mathbb{R}} e^{-j\omega t}dt = 2\pi\delta(\omega)$.

$\therefore\ e^{2+t}u(-t)+1 \leftrightarrow \frac{e^2}{1-j\omega} + 2\pi\delta(\omega)$.

(2) [$e^{-\alpha|t|} \leftrightarrow \int_{-\infty}^{0} e^{(\alpha-j\omega)t}dt + \int_0^{+\infty} e^{-(\alpha+j\omega)t}dt = \frac{1}{\alpha-j\omega}+\frac{1}{\alpha+j\omega}$

$= \frac{2\alpha}{\alpha^2+\omega^2}$, $\mathcal{F}[\sin(\omega_0 t)] = \frac{1}{2j}\left(e^{j\omega_0 t}-e^{-j\omega_0 t}\right)$,

$x(t)e^{j\omega_0 t} = \frac{1}{2\pi}\int_{\mathbb{R}} X(\underbrace{\omega}_{\omega'-\omega_0}) e^{j\underbrace{(\omega+\omega_0)}_{\omega'}} d\underbrace{(\omega+\omega_0)}_{\omega'} \leftrightarrow X(\omega-\omega_0)$]

$\Rightarrow e^{-3|t|}\sin 2t \leftrightarrow \left[\left.\frac{6}{9+\omega^2}\right|_{\omega=\omega-2} - \frac{6}{9+(\omega+2)^2}\right]/(2j)$

$= 3j\left[\frac{1}{9+(\omega+2)^2} - \frac{1}{9+(\omega-2)^2}\right]$.

2. 解：(3) $[e^{-\alpha t}u(t) \leftrightarrow \int_0^{+\infty} e^{-(\alpha+j\omega)t}dt = \frac{1}{\alpha+j\omega}$,

$\cos(\omega_0 t) = \frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t})$,

$x(t)e^{+j\omega_0 t} = \frac{1}{2\pi}\int_{\mathbb{R}} \underbrace{X(\omega)}_{\omega'-\omega_0} e^{j\overbrace{(\omega+\omega_0)}^{\omega'}t} d\underbrace{(\omega+\omega_0)}_{\omega'} \leftrightarrow X(\omega-\omega_0).$]

$\therefore \mathcal{F}[e^{-\alpha t}u(t)\cdot\cos(\omega_0 t)] = \left[\frac{1}{\alpha+j(\omega-\omega_0)} + \frac{1}{\alpha+j(\omega+\omega_0)}\right]/2.$

(4) $[\sin(\omega_0 t) = \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})$,

$\frac{d}{d\omega}X(\omega) = \cancel{\frac{1}{2\pi}}\int_{\mathbb{R}} x(t)(-jt)e^{-j\omega t}dt \leftrightarrow -jt\,x(t)$

$\Rightarrow t\,x(t) \leftrightarrow j\frac{d}{d\omega}X(\omega).$]

$t\,e^{-2t}u(t) \leftrightarrow j\frac{d}{d\omega}\left(\frac{1}{2+j\omega}\right) = \frac{1}{(2+j\omega)^2}$

$\Rightarrow \mathcal{F}[te^{-2t}u(t)\cdot\sin 4t] = \frac{1}{2j}\cdot\left[\frac{1}{(2+j(\omega-4))^2} - \frac{1}{(2+j(\omega+4))^2}\right].$

3. 解：(1)

$F(\omega) = |F(\omega)|e^{j\angle F(\omega)} = \begin{cases} Ae^{j\omega t_0}, & |\omega|<\omega_0, \\ 0, & |\omega|>\omega_0. \end{cases}$

$\Rightarrow \mathcal{F}^{-1}[F(\omega)] = \frac{1}{2\pi}\int_{\mathbb{R}} F(\omega)e^{j\omega t}d\omega = \frac{A}{2\pi}\cdot\int_{-\omega_0}^{\omega_0} e^{j\omega(t+t_0)}d\omega$

$= \begin{cases} \frac{A}{2\pi}\cdot\frac{2j\sin(\omega_0(t+t_0))}{j(t+t_0)} = \frac{A\sin(\omega_0(t+t_0))}{\pi(t+t_0)}, & t\neq -t_0. \\ \frac{A\omega_0}{\pi}, & t=-t_0. \end{cases}$

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3.(2) 解: $F(\omega) = |F(\omega)| e^{j\angle F(\omega)} = \begin{cases} -jA, & \omega\in(-\omega_0,0), \\ jA, & \omega\in(0,\omega_0), \\ 0, & |\omega|>\omega_0. \end{cases}$

$$\Rightarrow \mathcal{F}^{-1}[F(\omega)] := \frac{1}{2\pi}\int_{\mathbb{R}} F(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\omega_0}^{0} -jAe^{j\omega t}\,d\omega + \frac{1}{2\pi}\int_{0}^{\omega_0} jAe^{j\omega t}\,d\omega$$

$$= \frac{-jA}{2\pi}\frac{1-e^{-j\omega_0 t}}{j\cancel{\omega_0}t} + \frac{jA}{2\pi}\frac{e^{j\omega_0 t}-1}{j\cancel{\omega_0}t}.$$

$[\, C_{2x} = c^2 - s^2 = 2c^2 - 1 = 1 - 2s^2 \,]$

$$= \frac{A}{2\pi\cancel{\omega_0}t}\left[e^{-j\omega_0 t} + e^{j\omega_0 t} - 2\right] = \frac{A}{\pi\cancel{\omega_0}t}\left[\cos(\omega_0 t) - 1\right],\quad t\neq 0$$

$$= \begin{cases} \dfrac{-2A}{\pi\cancel{\omega_0}t}\sin^2\dfrac{\omega_0 t}{2}, & t\neq 0, \\ 0, & t=0. \end{cases}$$

4. 解: (1) $f(at) \overset{a\neq 0}{\longleftrightarrow} \int_{\mathbb{R}} f(\underbrace{at}_{\tau})e^{-j\omega t}\,dt = \int_{\mathbb{R}} f(\tau)e^{-j\frac{\omega}{a}\tau}\frac{d\tau}{|a|} = \frac{1}{|a|}F(\frac{\omega}{a}).$

$\therefore f(2t) \longleftrightarrow \frac{1}{2}F(\frac{\omega}{2}).$

$[\, \frac{d}{d\omega}F(\omega) = \int_{\mathbb{R}} f(t)\cdot(-jt)e^{-j\omega t}\,dt \Leftrightarrow -jtf(t)$

$\Rightarrow tf(t) \Leftrightarrow j\frac{d}{d\omega}F(\omega).\,]$

$\Rightarrow tf(2t) \longleftrightarrow \frac{j}{2}\frac{d}{d\omega}F(\frac{\omega}{2}) = \frac{j}{4}F'(\frac{\omega}{2})$, where $F'(\omega) := \left.\frac{dF(\omega)}{d\omega}\right|_{\omega=\omega}.$

(2) $\frac{d}{dt}f(t) = \frac{d}{dt}\cdot\frac{1}{2\pi}\int_{\mathbb{R}} F(\omega)e^{j\omega t}\,d\omega = \frac{1}{2\pi}\int_{\mathbb{R}} j\omega F(\omega)e^{j\omega t}\,d\omega \leftrightarrow j\omega F(\omega).$

$\therefore t\frac{df(t)}{dt} \longleftrightarrow j\frac{d}{d\omega}[\, j\omega F(\omega)\,] = -F(\omega) - \omega F'(\omega).$

(3) $f(-2t) \leftrightarrow \frac{1}{2}F(-\frac{\omega}{2})$, $\Rightarrow tf(-2t) = j\frac{d}{d\omega}[\,\frac{1}{2}F(-\frac{\omega}{2})\,] = -\frac{j}{4}F'(-\frac{\omega}{2}).$

$\therefore (t-2)f(2t) \longleftrightarrow -\frac{j}{4}F'(\frac{-\omega}{2}) - F(-\frac{\omega}{2}).$

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4 (d) 解: (d) $tf(t) \leftrightarrow jF'(\omega)$.

$[f(t+t_0) \leftrightarrow \int_{\mathbb{R}} f(t+t_0)e^{-j\omega(t+t_0)}d(t+t_0)\cdot e^{j\omega t_0} = e^{j\omega t_0}\cdot F(\omega)]$

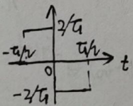
$\therefore\ (1+t)f(1+t) \leftrightarrow je^{j\omega}F'(\omega)$

$\therefore\ (1-t)f(1-t) \leftrightarrow je^{-j\omega}F'(-\omega)$, where $F'(-\omega) := \left.\frac{dF(\omega)}{d\omega}\right|_{\omega=-\omega}$.

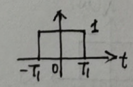
(e) $f(6+t) \leftrightarrow e^{j6\omega}F(\omega) \Rightarrow f(6-2t) \leftrightarrow \frac{1}{2}e^{j6\cdot\frac{\omega}{-2}}F(-\frac{\omega}{2})$.

5. 解: $f(t) := x(t)\cdot\cos\omega_0 t$, $x(t) := \begin{cases} 1-\frac{2}{\tau_1}|t|, & |t|<\tau_1/2, \\ 0, & |t|>\tau_1/2. \end{cases}$

$\Rightarrow X(\omega) := \mathcal{F}[x(t)] = \int_{-\tau_1/2}^{\tau_1/2}\left(1-\frac{2}{\tau_1}|t|\right)e^{-j\omega t}dt = \frac{8\sin^2(\frac{\tau_1}{4}\omega)}{\omega^2\tau_1} = \frac{\tau_1}{2}Sa^2(\frac{\tau_1}{4}\omega)$.

[$x'(t)$: 

$= \frac{2}{\tau_1}\left(x_0(t+\frac{\tau_1}{4}) - x_0(t-\frac{\tau_1}{4})\right)$,

$x_0(t)$: 

$\overset{X_0(\omega):=}{\longleftrightarrow} \int_{-T_1}^{T_1} e^{-j\omega t}dt = \frac{-2j\sin(\omega T_1)}{-j\omega} = \frac{2}{\omega}\sin(\omega T_1)$, $T_1 = \tau_1/4$

$\Rightarrow x'(t) \leftrightarrow j\omega X(\omega) = \frac{2}{\tau_1}\cdot\frac{2}{\omega}\sin(\omega\frac{\tau_1}{4})\cdot(e^{j\omega\frac{\tau_1}{4}} - e^{-j\omega\frac{\tau_1}{4}})$

$= \begin{cases} \frac{8j}{\omega\tau_1}\sin^2(\omega\frac{\tau_1}{4}), & \omega\neq 0, \\ \int_{\mathbb{R}} x'(t)dt = 0, & \omega = 0. \end{cases}$

$\Rightarrow x(t) = \int_{-\infty}^{t} x'(t)dt \leftrightarrow \frac{\mathcal{F}[x'(t)]}{j\omega} + \pi\mathcal{F}[x'(t)]|_{\omega=0}\delta(\omega) = \frac{8\sin^2(\frac{\tau_1}{4}\omega)}{\omega^2\tau_1}$, $\omega\neq 0$, $\because$

$X(\omega)|_{\omega=0} = \int_{\mathbb{R}} x(t)dt = \tau_1/2$.]

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$$\therefore f(t) = x(t)\cdot\frac{1}{2}\left(e^{j\omega_0 t}+e^{-j\omega_0 t}\right) \leftrightarrow \frac{1}{2}\left[X(\omega-\omega_0)+X(\omega+\omega_0)\right]$$

$$\left[\; x(t)e^{j\omega_0 t} = \frac{1}{2\pi}\int_{\mathbb{R}} \underbrace{X(\omega)}_{\omega'-\omega_0} e^{j\overbrace{(\omega+\omega_0)}^{\omega'}t} d\underbrace{(\omega+\omega_0)}_{\omega'} \leftrightarrow X(\omega-\omega_0)\;\right]$$

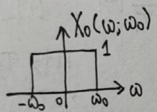
$$= \frac{\tau_1}{4}\left[Sa^2\left(\frac{\tau_1}{4}(\omega-\omega_0)\right)+Sa^2\left(\frac{\tau_1}{4}(\omega+\omega_0)\right)\right]; \cancel{\omega\neq 0},$$

$$\cancel{F(\omega)}\ \lim_{\omega\to 0^+} F(\omega) =$$

6. 解：$\left[\; X_1(\omega) * X_2(\omega) \leftrightarrow \frac{1}{2\pi}\int_{\mathbb{R}} e^{j\omega t} d\omega \int_{\mathbb{R}} X_1(\xi)X_2(\omega-\xi)d\xi \right.$

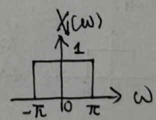
$$= \frac{1}{2\pi}\int_{\mathbb{R}} X_1(\xi)e^{j\xi t}d\xi \int_{\mathbb{R}} X_2(\omega-\xi)e^{j(\omega-\xi)t} d(\omega-\xi) = 2\pi\, x_1(t)x_2(t)$$

$$\left. \therefore\ x_1(t)\cdot x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega). \;\right]$$

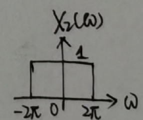
[

$$\leftrightarrow \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega t}d\omega = x_0(t;\omega_0) := \begin{cases} \dfrac{2j\sin(\omega_0 t)}{2\pi j t} = \dfrac{\sin(\omega_0 t)}{\pi t}, & t\neq 0, \\ \dfrac{\omega_0}{\pi} = \lim\limits_{t\to 0} \dfrac{\sin(\omega_0 t)}{\pi t}, & t=0. \end{cases}$$

$$x(t-t_0) \leftrightarrow \int_{\mathbb{R}} x(t-t_0)e^{-j\omega(t-t_0)} d(t-t_0)\, e^{-j\omega t_0} = e^{-j\omega t_0}X(\omega).\ \left.\right]$$

$$x_1(t) := \frac{\sin(\pi t)}{\pi t} \leftrightarrow X_0(\omega,\pi) =$$ 

,

$$x_2(t) := \frac{\sin(2\pi(t-1))}{\pi(t-1)} \leftrightarrow X_0(\omega,2\pi)e^{-j\omega} = e^{-j\omega}\cdot$$ 

$$\Rightarrow x_1(t)x_2(t) \leftrightarrow \frac{1}{2\pi}X_1(\omega) * X_2(\omega) =: F(\omega) = \frac{1}{2\pi}\int_{\mathbb{R}} X_1(\xi^{\omega-\xi})X_2(\xi)d\xi$$

$$= \begin{cases} \dfrac{1}{2\pi}\displaystyle\int_{\omega-\pi}^{2\pi} e^{-j\omega}d\omega = \left[1-e^{-j(\omega-\pi)}\right]\frac{1}{j}(+j)/2\pi = \dfrac{j}{2\pi}\left(1+e^{-j\omega}\right), & \pi<\omega<3\pi, \\ \dfrac{1}{2\pi}\displaystyle\int_{\omega-\pi}^{\omega+\pi} e^{-j\xi}d\xi = \dfrac{j}{2\pi}\left[-e^{-j\omega}+e^{j\omega}\right] = 0, & -\pi<\omega<\pi, \\ \dfrac{-1}{2\pi}\displaystyle\int_{\omega-\pi}^{-2\pi} e^{-j\xi}d\xi = \dfrac{-j}{2\pi}\left[1+e^{-j\omega}\right], & -3\pi<\omega<\pi, \\ 0, & \text{otherwise}. \end{cases}$$

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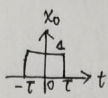
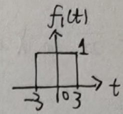
7. 解：(a) $\left[\ x_0(t;\tau) =\right.$  $\leftrightarrow X_0(\omega;\tau) = \int_{-\tau}^{\tau} e^{-j\omega t} dt = \begin{cases} \frac{-2j\sin(\omega\tau)}{-j\omega} = \frac{2}{\omega}\sin(\omega\tau), & \omega \neq 0, \\ 2\tau, & \omega = 0. \end{cases} \left.\right]$

$\left[\ X(\omega-\omega_0) \leftrightarrow \frac{1}{2\pi}\int_{\mathbb{R}} X(\omega-\omega_0) e^{j(\omega-\omega_0)t} d(\omega-\omega_0) e^{j\omega_0 t}\right.$

$\left. = e^{j\omega_0 t} x(t).\ \right]$

$\therefore \frac{2}{\omega}\sin[3\omega] =: F_1(\omega) \overset{= X_0(\omega,3)}{\leftrightarrow} x_0(t,3) =$ 

$\Rightarrow F(\omega) = F_1(\omega - 2\pi) \leftrightarrow e^{j2\pi t} f_1(t) = e^{j2\pi t} \cdot [u(t+3) - u(t-3)].$

(b) 解：$F(\omega) = \frac{1}{2}\left[e^{j(4\omega+\frac{\pi}{3})} + e^{-j(4\omega+\frac{\pi}{3})}\right] = \frac{1}{2} e^{j\frac{\pi}{3}} e^{j4\omega} + \frac{1}{2} e^{j(-\frac{\pi}{3})} e^{-j4\omega}.$

$\left[\ e^{j\omega t_0} \leftrightarrow \frac{1}{2\pi}\int_{\mathbb{R}} e^{j\omega(t+t_0)} d\omega = \frac{1}{2\pi} \cdot 2\pi \cdot \delta(t+t_0).\ \right]$

$\therefore F(\omega) \leftrightarrow \frac{1}{2} e^{j\frac{\pi}{3}} \delta(t+4) + \frac{1}{2} e^{-j\frac{\pi}{3}} \delta(t-4).$

(c) $\left[\ \delta(\omega) \leftrightarrow \frac{1}{2\pi}\int_{\mathbb{R}} \delta(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi}\ \right]$

$\left[\ X(\omega+\omega_0) \leftrightarrow \frac{1}{2\pi}\int_{\mathbb{R}} X(\omega+\omega_0) e^{j(\omega+\omega_0)t} d(\omega+\omega_0) \cdot e^{-j\omega_0 t} = e^{-j\omega_0 t} x(t)\ \right]$

$F(\omega) \leftrightarrow 2 \cdot \frac{1}{2\pi} \cdot (e^{j\phi t} - e^{-j\phi t}) + 3 \cdot \frac{1}{2\pi} \cdot (e^{j2\pi t} + e^{-j2\pi t})$

$= j\frac{2}{\pi} \sin\phi t + \frac{3}{\pi}\cos(2\pi t).$

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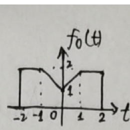
8. 解：$f(t) = f_0(t-1)$，$f_0(t) =$  .

(1) $\left[\ x(t+t_0) \leftrightarrow \int_{\mathbb{R}} x(t+t_0) e^{-j\omega(t+t_0)} d(t+t_0)\, e^{j\omega t_0} = e^{j\omega t_0} X(\omega)\ \right]$

$\left[\ x(t)\text{实}\quad x^*(t) = \frac{1}{2\pi}\int_{\mathbb{R}} X^*(\omega) e^{-j\omega t} d\omega = \frac{1}{2\pi}\int_{\mathbb{R}} X^*(-\omega) e^{j\omega t} d\omega \leftrightarrow X^*(-\omega),\right.$

$\left. x(-t) = \frac{1}{2\pi}\int_{\mathbb{R}} X(\omega) e^{-j\omega t} d\omega = \frac{1}{2\pi}\int_{\mathbb{R}} X(-\omega) e^{j\omega t} d\omega \leftrightarrow X(-\omega)\ \right]$

$\left[\ x\text{实} \overset{\text{def}}{\Longleftrightarrow} x(t) = x^*(t) \Leftrightarrow X(\omega) = X^*(-\omega)\right.$ i.e. $X(\omega)$ 共轭对称，

$\left. x\text{偶} \overset{\text{def}}{\Longleftrightarrow} x(t) = x(-t) \Leftrightarrow X(\omega) = X(-\omega)\right.$ i.e. $X(\omega)$ 偶.]

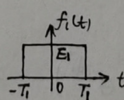
[x 实且偶 $\Leftrightarrow X(\omega) = X^*(-\omega) = X(-\omega) \Leftrightarrow X(\omega)$ 实且偶]

$$\left.\begin{array}{l} f(t) = f_0(t-1) \Rightarrow F(\omega) = F_0(\omega) e^{-j\omega} \\ f_0(t)\text{ 实且偶} \Rightarrow F_0(\omega)\text{ 实且偶} \end{array}\right\} \Rightarrow \operatorname{Arg} F(\omega) = \operatorname{Arg} F_0(\omega) + \operatorname{Arg} e^{-j\omega} = -\omega.$$

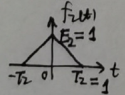
(2) $\left[\ x\text{实} \overset{\text{def}}{\Leftrightarrow} x(t) = x^*(t) \Leftrightarrow X(\omega) = X^*(-\omega)\ \right]$

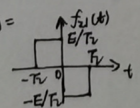
$\left[\ \overset{\Rightarrow}{\vee}\ \operatorname{Re}\{X(\omega)\} = \frac{1}{2}[X(\omega) + X^*(\omega)] \leftrightarrow \frac{1}{2}[x(t) + x^*(-t)] \overset{x\text{偶实}}{=\!=} x_e(t) := \frac{x(t) + x(-t)}{2}.\ \right]$

$\operatorname{Re}\{F(\omega)\} = \operatorname{Re}\{F_0(\omega) e^{-j\omega}\} \overset{F_0\text{实}}{=\!=} F_0(\omega)\cos\omega$.

$f_0(t) = f_1(t) - f_2(t)$，$f_1(t) =$  $\leftrightarrow F_1(\omega) = \int_{-T_1}^{T_1} E_1 e^{-j\omega t} dt$

$= \frac{E_1}{-j\omega}(-2j\sin(\omega T_1)) = \frac{2}{\omega} E_1 \sin(\omega T_1)$，$E_1 = 2$，$T_1 = 2$，

$f_2(t) =$  $= \int_{-\infty}^{t} f_{21}(\tau) d\tau \leftrightarrow \frac{F_{21}(\omega)}{j\omega} + \pi F_{21}(0)\delta(\omega) =: F_2(\omega)$,

$f_{21}(t) := f_2'(t) =$  $\leftrightarrow F_{21}(\omega) = \frac{2}{\omega}\frac{E_2}{T_2}\sin\left(\frac{T_2}{2}\omega\right)\cdot\left(e^{+j\omega\frac{T_2}{2}} - e^{-j\omega\frac{T_2}{2}}\right)$

$$= \begin{cases} j\frac{4E_2}{\omega T_2}\sin^2\left(\frac{T_2}{2}\omega\right), & \omega \neq 0, \\ 0, & \omega = 0. \end{cases}$$]

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$$\left[\Rightarrow F_2(\omega)=\begin{cases}\dfrac{4E}{\omega^2 T_2}\sin^2\left(\dfrac{T_2}{2}\omega\right)=T_2E_2\,Sa^2\left(\dfrac{T_2}{2}\omega\right), & \omega\neq 0,\\ E_2T_2, & \omega=0,\end{cases}\quad E_2=1,\ T_2=1.\right]$$

$$F_0(\omega)=F_1(\omega)-F_2(\omega)=\begin{cases}\dfrac{4}{\omega}\sin(2\omega)-Sa^2\left(\dfrac{\omega}{2}\right), & \omega\neq 0,\\ F_0(0+0)=8-1=7, & \omega=0.\end{cases}$$

$$\Rightarrow \mathrm{Re}\{F(\omega)\}=F_0(\omega)\cdot\cos\omega=\left[\frac{4}{\omega}\sin(2\omega)-Sa^2\left(\frac{\omega}{2}\right)\right]\cos\omega.$$

8. 解 (3) $F(0)=\int_{\mathbb{R}} f(t)dt = 4\times 2-1=7.$

(4) $\int_{\mathbb{R}} F(\omega)d\omega = 2\pi\cdot\frac{1}{2\pi}\int_{\mathbb{R}} F(\omega)e^{j\omega t}d\omega\Big|_{t=0} = 2\pi f(0) = 2\pi.$

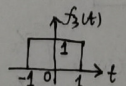
(5) $\int_{\mathbb{R}} |F(\omega)|^2 d\omega = 2\pi\int_{\mathbb{R}}|f(t)|^2dt = 2\pi\cdot[4\times 2]+2\pi\int_0^2|f(t)|^2dt$

$= 16\pi + 2\pi\int_0^2 |f_0(t-1)|^2 d(t-1) = 16\pi + 2\cdot 2\pi\int_0^1 \underbrace{f_0(t)}_{=t+1} d(t+1) =$

$= 16\pi + 4\pi\cdot\frac{1}{2}(4-1) = 22\pi.$

(6) 原式 $=\int_{\mathbb{R}} F(\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega = 2\pi\cdot\frac{1}{2\pi}\cdot\int_{\mathbb{R}}F(\omega)\frac{2\sin\omega}{\omega}e^{j\omega t}d\omega\Big|_{t=2}$

$= 2\pi\cdot\mathcal{F}^{-1}\left[F(\omega)\cdot\frac{2\sin\omega}{\omega}\right]\Big|_{t=2} = 2\pi\cdot[f(t)*f_3(t)]_{t=2},$

$\left[f_3(t)=\mathcal{F}^{-1}\left[\frac{2\sin\omega}{\omega}\right].=\right.$  $\left.\right].$

$[f(t)*f_3(t)]_{t=2} := \int_{\mathbb{R}} f(\tau)f_3(2-\tau)d\tau = \int_1^3 f(t)dt = \frac{1+2}{2}+2=\frac{7}{2}.$

$\Rightarrow$ 原式 $= 2\pi\cdot\frac{7}{2}=7\pi.$

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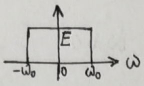
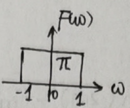
9. 解：(a)  $\leftrightarrow \frac{E}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega t}\,d\omega = \frac{E}{2\pi j t}\cdot 2j\sin(\omega_0 t) = \begin{cases} \frac{E\omega_0}{\pi}, & t=0, \\ \frac{E\sin(\omega_0 t)}{\pi t}, & t\neq 0, \end{cases}$

$\Rightarrow f(t) := \frac{\sin t}{t} \leftrightarrow F(\omega) =$ 

$\Rightarrow \int_{\mathbb{R}} |f(t)|^2 dt = \frac{1}{2\pi}\int_{\mathbb{R}} |F(\omega)|^2 d\omega = \frac{1}{2\pi}\cdot\pi^2\cdot 2 = \pi.$

(b) $[\, e^{-\alpha|t|} \overset{\alpha>0}{\leftrightarrow} \int_{-\infty}^{0} e^{(\alpha-j\omega)t}dt + \int_0^{+\infty} e^{-(\alpha+j\omega)t}dt = \frac{1}{\alpha-j\omega} + \frac{1}{\alpha+j\omega} = \frac{2\alpha}{\alpha^2+\omega^2}\,]$

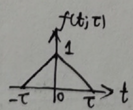
$[\, f(t) = \frac{1}{1+t^2} \leftrightarrow F(\omega) = \frac{1}{2} - \,]_{\times}$. $[\, \frac{2\alpha}{\alpha^2+t^2} \leftrightarrow 2\pi e^{-\alpha|\omega|}\,]$

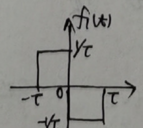
$[\, X(t) = \int_{\mathbb{R}} x(\tau)e^{-jt\tau}d\tau = 2\pi\cdot\frac{1}{2\pi}\int_{\mathbb{R}} x(-\omega)e^{+jt\omega}d\omega \leftrightarrow 2\pi x(-\omega)\,]$

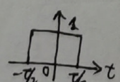
$\therefore f(t) = \frac{1}{1+t^2} \leftrightarrow F(\omega) = \pi e^{-|\omega|}$.

$\Rightarrow \int_{\mathbb{R}} |f(t)|^2 dt = \frac{1}{2\pi}\int_{\mathbb{R}} |F(\omega)|^2 d\omega = \frac{\pi^2}{2\pi}\int_{\mathbb{R}} e^{-2|\omega|}d\omega$

$= \pi\int_0^{+\infty} e^{-2\omega}d\omega = \pi\cdot\frac{1}{+2} = \pi/2.$

10. 解：$f_0(t;\tau) =$  $\Rightarrow \int_{-\infty}^{t} f_1(t)\,dt \leftrightarrow \frac{F_1(\omega)}{j\omega} + \pi F_1(0)\delta(\omega) =: F_0(\omega;\tau)$

其 $f_1(t) := f_0'(t;\tau) =$  $= \frac{1}{\tau}[\, f_2(t+\frac{\tau}{2}) - f_2(t-\frac{\tau}{2})\,] \leftrightarrow F_1(\omega)$.

$f_2(t) =$  $\leftrightarrow F_2(\omega) = \int_{-\tau/2}^{\tau/2} e^{-j\omega t}dt = \frac{-2j\sin(\frac{\tau}{2}\omega)}{-j\omega} = \begin{cases} \frac{2}{\omega}\sin(\frac{\tau}{2}\omega), & \omega\neq 0, \\ \tau, & \omega=0. \end{cases}$

$\Rightarrow F_1(\omega) = \frac{2}{\omega\tau}\sin(\frac{\tau}{2}\omega)\cdot[\, e^{j\omega\frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}}\,] = \begin{cases} \frac{j4}{\omega\tau}\sin^2(\frac{\tau}{2}\omega), & \omega\neq 0, \\ \int_{\mathbb{R}} f_1(t)dt = 0, & \omega = 0. \end{cases}$

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$$\Rightarrow F_0(\omega) = \begin{cases} \frac{4}{\omega^2\tau}\sin^2(\frac{\tau}{2}\omega) = \tau Sa^2(\frac{\tau}{2}\omega), & \omega \neq 0, \\ \tau = \lim_{\omega\to 0^+} F_0(\omega), & \omega = 0. \end{cases} \quad F_0(\omega;\tau) \leftrightarrow f_0(t;\tau).$$

$$[\; x(\omega) = \frac{1}{2\pi}\int_{\mathbb{R}} X(\xi)e^{j\xi\omega}d\xi \overset{\xi=-t}{=} \frac{1}{2\pi}\cdot\int_{\mathbb{R}} X(-t)e^{-j\omega t}dt \leftrightarrow \frac{X(-t)}{2\pi} \;]$$

$$\therefore\; f_0(\omega;\tau) \leftrightarrow \frac{1}{2\pi}F_0(-t;\tau).$$

$$[\; X(\omega+\omega_0) = \int_{\mathbb{R}} x(t)e^{-j(\omega+\omega_0)t}dt = \int_{\mathbb{R}}[x(t)\cdot e^{-j\omega_0 t}]e^{-j\omega t}dt \leftrightarrow e^{-j\omega_0 t}x(t) \;]$$

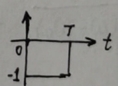
$$\Rightarrow F(\omega) = f_0(\omega-\omega_0;\omega_1) + f_0(\omega+\omega_0;\omega_1) \leftrightarrow \frac{F_0(-t;\omega_1)}{2\pi}[e^{j\omega_0 t}+e^{-j\omega_0 t}]$$

$$= \frac{1}{\pi}\cdot\cos(\omega_0 t)\cdot\omega_1 Sa^2(\frac{\omega_1}{2}t).$$

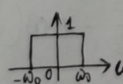
chapter 4-3

1. 解: (a) $H_1(\omega) \leftrightarrow h_1(t) = \frac{1}{2\pi}\int_{\mathbb{R}} H_1(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-1}^{1} e^{j\omega(t-t_0)}d\omega$

$$= \frac{2j\sin(t-t_0)}{j2\pi(t-t_0)} = \frac{1}{\pi}\cdot Sa(t-t_0).$$

$u_2(t) = [u(t-T)-u(t)] * h_1(t) =$  $* \frac{1}{\pi}Sa(t-t_0)$

$$= \frac{1}{\pi}\int_{t-T}^{T} -\frac{\sin(t-t_0)}{t-t_0}dt.$$

(b)  $\leftrightarrow \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega t}d\omega = \frac{2j\sin(\omega_0 t)}{j2\pi t} = \frac{\sin(\omega_0 t)}{\pi t}$

$$\Rightarrow u_1(t) = 2\pi\frac{\sin(\frac{1}{2}t)}{\pi t} \leftrightarrow U_1(\omega) = \begin{cases} 2\pi, & |\omega|<1/2, \\ 0, & \text{otherwise}. \end{cases}$$

$[\; 1 \leftrightarrow 2\pi\delta(\omega),\; \delta(t) \leftrightarrow 1,\; x(t)e^{j\omega_0 t} \leftrightarrow X(\omega-\omega_0). \;]$

$$\Rightarrow U_2(\omega) = U_1(\omega)[e^{-j\omega T}-1]\cdot H_1(\omega) = \begin{cases} 2\pi(e^{-j\omega T}-1)e^{-j\omega t_0}, & |\omega|<1/2, \\ 0, & \text{otherwise}. \end{cases}$$

~~$u_2(t)$~~ ~~$\mathcal{F}^{-1}[U_2(\omega)]$~~ ~~$2\pi(\delta(t-t_0-T) - \delta(t-t_0$~~

3 Chapter 4-3

$$\Rightarrow F_0(\omega) = F_0(\omega;\tau) \begin{cases} \dfrac{4}{\omega^2\tau}\sin^2\left(\dfrac{\tau}{2}\omega\right) = \tau Sa^2\left(\dfrac{\tau}{2}\omega\right), & \omega \neq 0, \\ \tau = \lim\limits_{\omega\to 0^+} F_0(\omega), & \omega = 0. \end{cases} \leftrightarrow f_0(t;\tau).$$

$$\left[\ x(\omega) = \frac{1}{2\pi}\int_{\mathbb{R}} X(\xi)e^{j\xi\omega}\,d\xi \overset{\xi=-t}{=} \frac{1}{2\pi}\cdot\int_{\mathbb{R}} X(-t)e^{-j\omega t}\,dt \leftrightarrow \frac{X(-t)}{2\pi}\ \right]$$

$$\therefore f_0(\omega;\tau) \leftrightarrow \frac{1}{2\pi}F_0(-t;\tau).$$

$$\left[\ X(\omega+\omega_0) = \int_{\mathbb{R}} x(t)e^{-j(\omega+\omega_0)t}\,dt = \int_{\mathbb{R}}\left[x(t)\cdot e^{-j\omega_0 t}\right]e^{-j\omega t}\,dt \leftrightarrow e^{-j\omega_0 t}x(t)\ \right]$$

$$\Rightarrow F(\omega) = f_0(\omega-\omega_0;\omega_1) + f_0(\omega+\omega_0;\omega_1) \leftrightarrow \frac{F_0(-t;\omega_1)}{2\pi}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right]$$

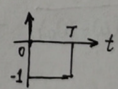
$$= \frac{1}{\pi}\cdot\cos(\omega_0 t)\cdot\omega_1 Sa^2\left(\frac{\omega_1}{2}t\right).$$

chapter 4-3

1. 解: (a) $H_1(\omega) \leftrightarrow h_1(t) = \dfrac{1}{2\pi}\displaystyle\int_{\mathbb{R}} H_1(\omega)e^{j\omega t}\,d\omega = \dfrac{1}{2\pi}\int_{-1}^{1} e^{j\omega(t-t_0)}\,d\omega$

$$= \frac{2j\sin(t-t_0)}{j2\pi(t-t_0)} = \frac{1}{\pi}\cdot Sa(t-t_0).$$

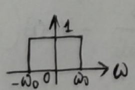
$$u_2(t) = [u(t-T)-u(t)] * h_1(t) =$$



$$* \frac{1}{\pi}Sa(t-t_0)$$

$$= \frac{1}{\pi}\int_{t-T}^{T} -\frac{\sin(t-t_0)}{t-t_0}\,dt.$$

(b)



$$\leftrightarrow \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega t}\,d\omega = \frac{2j\sin(\omega_0 t)}{j2\pi t} = \frac{\sin(\omega_0 t)}{\pi t}$$

$$\Rightarrow u_1(t) = 2\pi\frac{\sin(\frac{1}{2}t)}{\pi t} \leftrightarrow U_1(\omega) = \begin{cases} 2\pi, & |\omega| < \frac{1}{2}, \\ 0, & \text{otherwise}. \end{cases}$$

$[\ 1 \leftrightarrow 2\pi\delta(\omega),\ \delta(t) \leftrightarrow 1,\ x(t)e^{j\omega_0 t} \leftrightarrow X(\omega-\omega_0).\ ]$

$$\Rightarrow U_2(\omega) = U_1(\omega)[e^{-j\omega T}-1]\cdot H_1(\omega) = \begin{cases} 2\pi(e^{-j\omega T}-1)e^{-j\omega t_0}, & |\omega| < \frac{1}{2}, \\ 0, & \text{otherwise}. \end{cases}$$

$\cancel{\Rightarrow}$ $u_2(t)$ $\cancel{=}$ $\mathcal{F}^{-1}[U_2(\omega)]$ $\cancel{=}$ $2\pi(\delta(t-t_0-T) - \delta(t-t_0$

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$$\Rightarrow u_2(t) = \frac{1}{2\pi}\int_{\mathbb{R}} U_2(\omega) e^{j\omega t} d\omega = \int_{-1/2}^{1/2} \left[e^{-j\omega(T+t_0-t)} - e^{-j\omega(t_0-t)} \right] d\omega$$

$$= \frac{-2j\sin\frac{T+t_0-t}{2}}{-j(T+t_0-t)} - \frac{-2j\sin\left(\frac{t_0-t}{2}\right)}{-j(t_0-t)} = Sa\left(\frac{T+t_0-t}{2}\right) - Sa\left(\frac{t_0-t}{2}\right).$$

2. 解：(1) $x(t) = e^{jt} \leftrightarrow X(\omega) = \int_{\mathbb{R}} e^{-j(\omega-1)t} dt = 2\pi\delta(\omega-1)$.

$$\Rightarrow Y(\omega) = X(\omega)H(\omega) = 2\pi\delta(\omega-1)\cdot(-2j\omega) = -j4\pi\delta(\omega-1)$$

$$\Rightarrow y_{zs}(t) = \mathcal{F}^{-1}[Y(\omega)] = \frac{1}{2\pi}\int_{\mathbb{R}} -j4\pi e^{j\omega t}\delta(\omega-1)\,d\omega$$

$$= -2je^{jt}.$$

(2) $\delta(t) \leftrightarrow 1$, $\Rightarrow u(t) = \int_{-\infty}^{t} \delta(\tau)d\tau \leftrightarrow \frac{1}{j\omega} + \pi\delta(\omega) =: U(\omega)$

$$\Rightarrow x(t) = \frac{1}{2j}\cdot\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)\cdot u(t) \leftrightarrow X(\omega) = \frac{1}{2j}\left[U(\omega-\omega_0) - U(\omega+\omega_0)\right]$$

$$= -\frac{1}{2(\omega-\omega_0)} + \frac{1}{2(\omega+\omega_0)} + \frac{\pi}{2j}\left(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right),$$

$$\left[\ x(t)e^{j\omega_0 t} \leftrightarrow \int_{\mathbb{R}} x(t)e^{-j(\omega-\omega_0)t}dt = X(\omega-\omega_0)\ \right]$$

$$\Rightarrow Y(\omega) = \mathcal{F}[y_{zs}(t)] = X(\omega)H(\omega) = \frac{j\omega}{\omega-\omega_0} - \frac{j\omega}{\omega+\omega_0} - \pi\omega\left(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right)$$

$$= j\frac{2\omega_0\omega}{\omega^2-\omega_0^2} - \pi\omega_0\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

$$\left[\ e^{-\alpha|t|} \leftrightarrow \int_{-\infty}^{0} e^{(\alpha-j\omega)t}dt + \int_{0}^{+\infty} e^{-(\alpha+j\omega)t}dt = \frac{1}{\alpha-j\omega} + \frac{1}{\alpha+j\omega} = \frac{2\alpha}{\alpha^2+\omega^2}\ \right]$$

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$$Y(\omega) = j\left[\frac{\omega_0}{\omega-\omega_0} + \frac{+\omega_0}{\omega+\omega_0}\right] + \pi\omega\left[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)\right].$$

$$\left[\cos(\omega_0 t)u(t) \cancel{\longleftrightarrow} \int_0^{+\infty}\cos(\omega_0 t)e^{-j\omega t}dt = \frac{1}{2}\int_0^{+\infty}\left[e^{-j(\omega-\omega_0)t} + e^{-j(\omega+\omega_0)t}\right]dt\right.$$

$$\cancel{=} \frac{1}{2}\cdot\left[\frac{\quad}{-j(\omega-\omega_0)}\quad\right]$$

$$\left[\cos(\omega_0 t)u(t)\right] \longleftrightarrow \frac{1/2}{j(\omega-\omega_0)} + \frac{\pi}{2}\delta(\omega-\omega_0) + \frac{1/2}{j(\omega+\omega_0)} + \frac{\pi}{2}\delta(\omega+\omega_0)$$

$$= \frac{\omega}{j(\omega^2-\omega_0^2)} + \frac{\pi}{2}\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right).$$

$$Y(\omega) = -2\omega_0\mathcal{F}\left[\cos(\omega_0 t)u(t)\right] \Rightarrow y_{zs}(t) = \mathcal{F}^{-1}\left[Y(\omega)\right] = -2\omega_0\cos(\omega_0 t)u(t).$$

3.2. 解: $\cancel{(1)}$ $Y(\omega) = -2j\omega X(\omega) \Rightarrow y_{zs}(t) = -2\frac{d}{dt}x(t)$

(1) $y_{zs}(t) = -2\frac{d}{dt}(e^{jt}) = -2je^{jt}$.

(2) $y_{zs}(t) = -2\frac{d}{dt}\left[\sin(\omega_0 t)u(t)\right] = -2\omega_0\cos(\omega_0 t)u(t)$.

3. 解: (1) $Y(\omega) = H(\omega)X(\omega) = \frac{-2}{6+j\omega} \longleftrightarrow -2e^{-6t}u(t)$.

$$\left[e^{-\alpha t}u(t) \overset{\alpha>0}{\longleftrightarrow} \int_0^{+\infty} e^{-(\alpha+j\omega)t}dt = +\frac{1}{\alpha+j\omega}\right]$$

(2) $X(\omega) = \frac{1}{2+j\omega} \longleftrightarrow x(t) = e^{-2t}u(t)$.

$$Y(\omega) = H(\omega)X(\omega) = -2j\omega X(\omega) \Rightarrow y_{zs}(t) = -2\frac{d}{dt}x(t)$$

$$\left[\frac{d}{dt}x(t) = \frac{d}{dt}\cdot\frac{1}{2\pi}\int_{\mathbb{R}} X(\omega)e^{+j\omega t}dt \longleftrightarrow j\omega X(\omega)\right]$$

$$\Rightarrow y_{zs}(t) = -2\frac{d}{dt}\left[e^{-2t}u(t)\right] = 4e^{-2t}u(t) - 2\delta(t).$$

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4. 解: ~~(a)~~ $x(t) = \sum_k a_k e^{jk\omega_0 t}$, $\omega_0 = \frac{2\pi}{T}$.

$\mathcal{F}[x(t)] =: X(\omega) = 2\pi \sum_k a_k \delta(\omega - k\omega_0) \Rightarrow Y(\omega) = H(\omega)X(\omega) = \cancel{2\pi \sum_k a_k}$

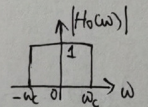
$= 2\pi \sum_k a_k H(k\omega_0)\delta(\omega - k\omega_0) \leftrightarrow y_{zs}(t) = \sum_k a_k H(k\omega_0) e^{jk\omega_0 t}$.

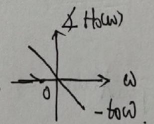
(a) $x(t) = \left[\frac{e^{j\theta}}{2} e^{j2\pi t} + \frac{1}{2} e^{-j\theta} e^{-j2\pi t}\right]$, $|\pm 2\pi| < 3\pi$.

$\Rightarrow y_{zs}(t) = \frac{1}{3\pi}\frac{d}{dt}[x(t)] = -\frac{1}{3\pi}\cdot 2\pi \cdot \sin(2\pi t + \theta)$.

(b) $x(t) \leftrightarrow X(\omega) = \begin{cases} \sim, & \omega = \pm 4\pi, \\ 0, & \text{otherwise.} \end{cases} \Rightarrow Y(\omega) = H(\omega)X(\omega) = 0$

$\Rightarrow y_{zs}(t) = 0$.

5. 解: $H_0(\omega) =$ 

, 

$= \begin{cases} e^{-j\omega t_0}, & |\omega| < \omega_c, \\ 0, & \text{otherwise.} \end{cases}$

$\leftrightarrow h_0(t) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{+j\omega(t-t_0)} d\omega = \frac{2j\sin[\omega_c(t-t_0)]}{j2\pi(t-t_0)} = \frac{\sin[\omega_c(t-t_0)]}{\pi(t-t_0)}$.

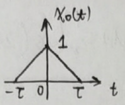
$X(\omega+\omega_0) \leftrightarrow \frac{1}{2\pi}\int_{\mathbb{R}} X(\omega+\omega_0) e^{j(\omega+\omega_0)t} d(\omega+\omega_0) e^{-j\omega_0 t} = x(t)e^{-j\omega_0 t}$.

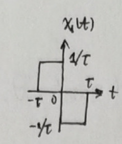
(1)

$H(\omega) = H_0(\omega+\omega_0) + H_0(\omega-\omega_0) \leftrightarrow \overset{h_0(t)}{\cancel{H(\omega)}} \cdot (e^{-j\omega_0 t} + e^{j\omega_0 t}) = 2\overset{h_0(t)}{\cancel{H(\omega)}}\cos(\omega_0 t)$

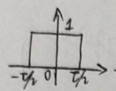
$= \frac{2}{\pi}\cos(\omega_0 t)\frac{\sin[\omega_c(t-t_0)]}{t-t_0} =: h(t)$

因 $h(t) \neq 0$ $(t<0)$，故这系统是非因果的. ⇒ not physically attainable.

(2) $x_0(t;\tau) =$  $= \int_{-\infty}^{t} x_1(\tau)d\tau \longleftrightarrow X_0(\omega) = \frac{X_1(\omega)}{j\omega} + \pi X_1(0)\delta(\omega)$,

$x_1(t) := \frac{d}{dt}x_0(t) =$  $= \frac{1}{\tau}\left(x_2(t+\frac{\tau}{2}) - x_2(t-\frac{\tau}{2})\right) \longleftrightarrow X_1(\omega)$,

$X_1(\omega) = \frac{1}{\tau}X_2(\omega)\cdot\left(e^{j\omega\frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}}\right) = j\frac{2}{\tau}X_2(\omega)\sin(\frac{\tau}{2}\omega)$,

$x_2(t) =$  $\longleftrightarrow X_2(\omega) = \int_{-\tau/2}^{\tau/2} e^{-j\omega t}dt = \frac{-2j\sin(\frac{\tau}{2}\omega)}{-j\omega} = \frac{2}{\omega}\sin(\frac{\tau}{2}\omega)$,

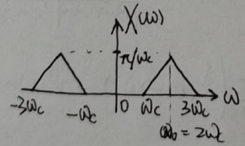
$$\Rightarrow X_1(\omega) = \begin{cases} j\frac{4}{\omega\tau}\sin^2(\frac{\tau}{2}\omega) \\ 0, \quad \omega=0 \end{cases} \Rightarrow X_0(\omega;\tau) = \frac{4}{\omega^2\tau}\sin^2(\frac{\tau}{2}\omega) = \tau Sa^2(\frac{\tau}{2}\omega),$$

$$[\ X(t) = \int_{\mathbb{R}} x(\tau)e^{-jt\tau}d\tau \ \underline{\underline{\tau=-\omega}}\ \int_{\mathbb{R}} x(-\omega)\,e^{j\omega t}dt \longleftrightarrow 2\pi x(-\omega)\ ]$$

$$\Rightarrow X_0(t;\omega_c) = \omega_c Sa^2(\frac{\omega_c}{2}t) \longleftrightarrow 2\pi x_0(-\omega;\omega_c) = 2\pi x_0(\omega;\omega_c).$$

$$\therefore x(t) = Sa^2(\frac{\omega_c}{2}t)\frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t}) \longleftrightarrow \frac{\pi}{\omega_c}\left[x_0(\omega-\omega_0;\omega_c) + x_0(\omega+\omega_0;\omega_c)\right] =: X(\omega),$$

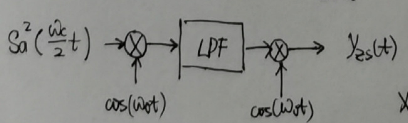
$(\omega_0 = 2\omega_c)$

$X(\omega) =$  $\Rightarrow Y(\omega) = X(\omega)H(\omega)$

$$Y(\omega) = \frac{\pi}{\omega_c}\left[x_0(\omega-\omega_0;\omega_c)e^{-j(\omega-\omega_0)t_0} + x_0(\omega+\omega_0;\omega_c)e^{-j(\omega+\omega_0)t_0}\right]$$

$$\longleftrightarrow y_{zs}(t) = \frac{\pi}{2\pi\omega_c}\left[X_0(t-t_0)e^{j\omega_0(t-t_0)}e^{j\omega_0 t_0} + X_0(t-t_0;\omega_c)e^{-j\omega_0(t-t_0)}e^{-j\omega_0 t_0}\right]$$

$$= X_0(t-t_0;\omega_c)\cdot\frac{2\cos(\omega_0 t)}{2\omega_c} = Sa^2(\frac{\omega_c}{2}(t-t_0))\cos(\omega_0 t).$$



X.

△ 物理解释？

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6. 解: $W(t) = g(t)\cos(\omega_c t)\cos(\omega_c t+\theta_c) = g(t)\cdot\frac{1}{2}\left(\cos(2\omega_c t+\theta_c)+\cos(\theta_c)\right)$

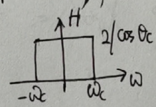
[$c_\alpha c_\beta = \frac{1}{2}[c(\alpha+\beta)+c(\alpha-\beta)]$]

$\Rightarrow W(t) = \frac{1}{2}g(t)\cos\theta_c + \frac{1}{2}g(t)\cos(2\omega_c t+\theta_c)$.

$\leftrightarrow \tilde{W}(\omega) = \frac{1}{2}G(\omega)\cos\theta_c + \frac{1}{4}[G(\omega-2\omega_c)e^{j\theta_c} + G(\omega+2\omega_c)e^{-j\theta_c}]$.

[$\cos(2\omega_c t+\theta_c) = \frac{1}{2}e^{j\theta_c}e^{j2\omega_c t} + \frac{1}{2}e^{-j\theta_c}e^{-j2\omega_c t}$.]

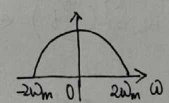
若 $G(\omega)=0$，($\forall|\omega|>\omega_c$)，且 θ_c 对 $\forall t$ 为同一个常数，则

通过 LPF: $H(\omega)=$ 

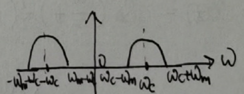
可恢复 $g(t)$. 否则，若

$G(\omega)$ 不是带限的或 θ_c 为时 t 变化，则未必能恢复.

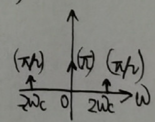
7. 解: $[g(t)+\cos(\omega_c t)]^2 = [g(t)]^2 + 2g(t)\cos(\omega_c t) + \frac{1}{2} + \frac{1}{2}\cos(2\omega_c t)$.

$[g(t)]^2 \leftrightarrow \frac{1}{2\pi}G(\omega)*G(\omega) =$ 

,

$2g(t)\cos(\omega_c t) \leftrightarrow G(\omega-\omega_c)+G(\omega+\omega_c) =$ 

,

$\frac{1}{2}+\frac{1}{2}\cos(2\omega_c t) \leftrightarrow \pi\delta(\omega) + \frac{\pi}{2}\left(\delta(\omega-2\omega_c)+\delta(\omega+2\omega_c)\right) =$ 

$\Rightarrow$ 要求: $\omega_c-\omega_m \geq 2\omega_m \Leftrightarrow \omega_c \geq 3\omega_m$

且 $2\omega_c \geq \omega_c+\omega_m \Leftrightarrow \omega_c \geq \omega_m$

且 $2\omega_m \leq \omega_L \leq \omega_c-\omega_m$，$\omega_c+\omega_m \leq \omega_H \leq 2\omega_c$

且 $A = 1/2$.

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8. 解: (a) $X_1(\omega) * X_2(\omega) = \int_{\mathbb{R}} X_1(\xi) X_2(\omega-\xi)\, d\xi$

$$\Longleftrightarrow \int_{\mathbb{R}} \frac{1}{2\pi} e^{j\omega t} d\omega \int_{\mathbb{R}} X_1(\xi) X_2(\omega-\xi)\, d\xi$$

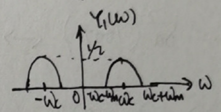
$$= \int_{\mathbb{R}} X_1(\xi) e^{j\xi t} d\xi \int_{\mathbb{R}} \frac{1}{2\pi} X_2(\omega-\xi)\, e^{j(\omega-\xi)t} d(\omega-\xi)$$

$$= 2\pi x_1(t)\cdot x_2(t), \quad \text{i.e.} \quad x_1(t)x_2(t) \longleftrightarrow \frac{1}{2\pi} X_1(\omega) * X_2(\omega).$$

(a)

$$Y_1(\omega) = \frac{1}{2\pi}\cdot G(\omega) * \frac{2\pi}{2}\left[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)\right]$$

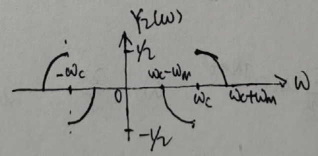
$$= \frac{1}{2}\left[G(\omega+\omega_c)+G(\omega-\omega_c)\right] =$$



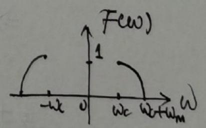
$$Y_2(\omega) = \frac{1}{2\pi}\cdot\left[\begin{cases} jG(\omega), & \omega>0, \\ -jG(\omega), & \omega<0 \end{cases}\right] * \frac{1}{2}\cdot\frac{2\pi}{2j}\left[\delta(\omega-\omega_c)-\delta(\omega+\omega_c)\right]$$

$$= \begin{cases} \frac{1}{2}G(\omega-\omega_c), & \omega>\omega_c, \\ -\frac{1}{2}G(\omega-\omega_c), & \omega<\omega_c \end{cases} + \begin{cases} -\frac{1}{2}G(\omega+\omega_c), & \omega>-\omega_c, \\ \frac{1}{2}G(\omega+\omega_c), & \omega<-\omega_c \end{cases}$$

$=$



$$\Rightarrow F(\omega) = Y_1(\omega)+Y_2(\omega) = \begin{cases} G(\omega-\omega_c), & \omega>\omega_c, \\ 0, & \text{otherwise} \\ G(\omega+\omega_c), & \omega<-\omega_c, \end{cases} =$$



(b) 可见，保留了上边带.

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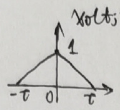
9. (1) $\omega_m = 4000\pi$, $\Rightarrow$ $\omega_s = 2\omega_m = 8000\pi$, $T_s = \frac{2\pi}{\omega_s} = \frac{1}{4000}$.

(2)

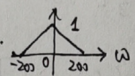


$\longleftrightarrow X_0(\omega;\tau) = \tau Sa^2(\frac{\tau}{2}\omega)$.

$\therefore \quad Sa^2(\frac{\tau}{2}\cancel{\omega}t) \longleftrightarrow \frac{2\pi}{\tau} x_0(\omega;\tau)$

$[\ X(t) = \int_{\mathbb{R}} x(\tau)e^{-jt\tau}d\tau \xlongequal{\tau=-\omega} \int_{\mathbb{R}} x(-\omega)e^{j\omega t}d\omega \longleftrightarrow 2\pi x(-\omega)\ ]$

$\therefore \quad Sa^2(100t) \longleftrightarrow \frac{\pi}{100}\cdot$



, $\Rightarrow \omega_m = 200$

$\Rightarrow \omega_s = 2\omega_m = 400 \Rightarrow T_s = \frac{2\pi}{\omega_s} = \frac{\pi}{200}$.

10. 解: $Y(\omega) = F_1(\omega)F_2(\omega) = 0$, $|\omega| > 1000\pi =: \omega_m$.

$y_s(t) = \sum_n y(nT)\delta(t-nT) \cancel{\leftrightarrow Y_s(\omega)} = y(t)\cdot\sum_n \delta(t-nT)$

$\longleftrightarrow \frac{1}{2\pi}\cdot Y(\omega) * [\ \frac{1}{T}\sum_n 2\pi\delta(\omega-n\omega_0)\ ] = \frac{1}{T}\sum_n Y(\omega-n\omega_0)$.

$[\ \sum_n \delta(t-nT) = \sum_n a_n e^{jn\omega_0 t}$, $\omega_0 = \frac{2\pi}{T}$, $a_n = \frac{1}{T}$

$\Rightarrow \sum_n \delta(t-nT) \longleftrightarrow \frac{1}{T}\cdot\sum_n 2\pi\delta(\omega-n\omega_0)\ ]$

要求 $\omega_0 \geqslant 2\omega_m \Rightarrow T \cancel{< \frac{1}{2f_m}} = \frac{2\pi}{\omega_0} \leqslant \frac{2\pi}{2\omega_m} = 1\times10^{-3}$.

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11. $e^{-\alpha t}u(t) \overset{\alpha>0}{\longleftrightarrow} \int_0^{+\infty} e^{-(\alpha+j\omega)t}\,dt = \frac{+1}{+(\alpha+j\omega)}, \quad \alpha>0$

(a) $X(\omega) = \frac{1}{1+j\omega} + \frac{1}{3+j\omega} = \frac{4+2j\omega}{(1+j\omega)(3+j\omega)},$

$Y(\omega) = \frac{2}{1+j\omega} + \frac{-2}{4+j\omega} = \frac{6}{(1+j\omega)(4+j\omega)}$

$\Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{6\cdot(3+j\omega)}{2(2+j\omega)(4+j\omega)} = \frac{3/2}{2+j\omega} + \frac{+3/2}{4+j\omega}$

(b) $H(\omega) \leftrightarrow h(t) = \frac{3}{2}(e^{-2t} + e^{-4t})u(t).$

(c) $\sum_k a_k \frac{d^k}{dt^k} y(t) = \sum_k b_k \frac{d^k}{dt^k} x(t)$

$\Rightarrow \sum_k a_k (j\omega)^k Y(\omega) = \sum_k b_k (j\omega)^k X(\omega)$

$\Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{\sum_k b_k (j\omega)^k}{\sum_k a_k (j\omega)^k} = \frac{9+3(j\omega)}{8+6(j\omega)+(j\omega)^2}.$

$\Rightarrow b_0 = 9,\ b_1 = 3,\ a_0 = 8,\ a_1 = 6,\ a_2 = 2$

$\Rightarrow$ ODE: $2\frac{d^2}{dt^2}y(t) + 6\frac{d}{dt}y(t) + 8y(t) = 3\frac{d}{dt}x(t) + 9x(t).$

12. 解: $z(t) \leftrightarrow Z(\omega) = \frac{1}{1+j\omega} + 3, \quad \int_{\mathbb{R}} x(\tau)z(t-\tau)\,d\tau = x(t) * z(t) \leftrightarrow X(\omega)Z(\omega).$

(a) $(j\omega+3)Y(\omega) = (Z(\omega)-1)X(\omega)$

$\Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{Z(\omega)-1}{j\omega+3} = \frac{1+2+2(j\omega)}{(j\omega+1)(j\omega+3)} = \frac{1/2}{j\omega+1} + \frac{+3/2}{j\omega+3}$

(b) $h(t) = \mathcal{F}^{-1}[H(\omega)] = \left(\frac{1}{2}e^{-t} + \frac{3}{2}e^{-3t}\right)u(t).$

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References