第 9 章作业

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摘　要:（ch9.2, Q7）双边 Laplace 变换的卷积性质 $x_1(t) * x_2(t) \leftrightarrow X_1(s) * X_2(s)$ 的 ROC 至少包含 $R_1 \cap R_2$，要求 $R_1 \cap R_2 \neq \phi$；当 $X_1(s), X_2(s)$ 发生零、极点抵消时，ROC 可能扩大.（ch9.2, Q13 另解）因果稳定 LTI 系统**频率响应**的定义和推导.

关键词: 词 1，词 2

Homework (Chapter 9)

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Abstract: Abstract.

Keywords: keyword 1, keyword

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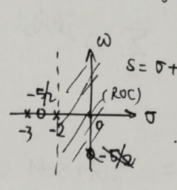
ICE2301 Homework (Chapter 9-1) 2022.05.08 (due)

1. 解: (a) $e^{-\alpha t}u(t) \leftrightarrow \int_{0^-}^{+\infty} e^{-(\alpha+s)t}\,dt = \left.\frac{e^{-(\alpha+s)t}}{-(\alpha+s)}\right|_{t=0^-}^{+\infty} = \frac{1}{\alpha+s}$, $Re[s] > Re[-\alpha]$.

$\Rightarrow X(s) = \frac{1}{s+2} + \frac{1}{s+3} = \frac{2s+5}{(s+2)(s+3)}$, $Re[s] > -2$.

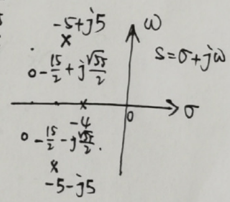


(b) $\cancel{e^{-\alpha t}}\sin(\omega_0 t)u(t) = \frac{1}{2j}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right]u(t) \leftrightarrow \left[\frac{1}{s-j\omega_0} - \frac{1}{s+j\omega_0}\right]\frac{1}{2j} = \frac{\omega_0}{s^2+\omega_0^2}$, $Re(s) > 0$.

$\Rightarrow X(s) = \frac{1}{s+4} + \frac{\cancel{s+5}\ 5}{(s+5)^2+25}$, $Re(s) > -4$

$= \frac{(s+\frac{15}{2})^2 + \frac{55}{4}}{(s+4)\left[(s+5)^2+25\right]} \Rightarrow$ $p_1 = -4$, $p_{2,3} = \cancel{-5\pm5j}, -5\pm j5$,

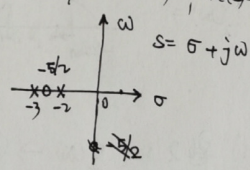
$z_{1,2} = -\frac{15}{2} \pm j\frac{\sqrt{55}}{2}$



(c) $e^{-\alpha t}u(-t) \leftrightarrow \int_{-\infty}^{0^+} e^{-(\alpha+s)t}\,dt = \left.\frac{e^{-(\alpha+s)t}}{-(\alpha+s)}\right|_{t=-\infty}^{0^-} = -1/(s+\alpha)$, $Re(s) < Re(-\alpha)$.

$\Rightarrow X(s) = -\frac{1}{s+2} - \frac{1}{s+3}$

$= \frac{-(2s+5)}{(s+2)(s+3)}$, $Re(s) < -3$.



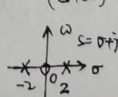
(d) $e^{-\alpha|t|} = e^{-\alpha t}u(t) + e^{\alpha t}u(-t) \leftrightarrow \frac{1}{s+\alpha} - \frac{1}{s-\alpha} = \frac{-2\alpha}{s^2-\alpha^2}$,

$Re(\alpha) > Re(s) > -Re(\alpha)$.

$\frac{d}{ds}X(s) = \frac{d}{ds}\int_{\mathbb{R}} x(t)e^{-st}\,dt \leftrightarrow -tx(t)$

$\Rightarrow te^{-\alpha|t|} \leftrightarrow -\frac{d}{ds}\left[\frac{-2\alpha}{s^2-\alpha^2}\right] = \frac{-1}{(s+\alpha)^2} + \frac{1}{(s-\alpha)^2} = \frac{2s(2\alpha)}{(s+\alpha)^2(s-\alpha)^2}$.

$\Rightarrow te^{-2|t|} \leftrightarrow \frac{-1}{(s+2)^2} + \frac{1}{(s-2)^2}$, $2 > Re(s) > -2$.

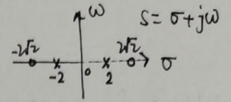


1. 解: (e) $x(t) = te^{-2t}u(t) - te^{2t}u(-t) = t\left(e^{-2t}u(t) - e^{2t}u(-t)\right)$.

$e^{-\alpha t}u(t) \leftrightarrow \int_{0^-}^{+\infty} e^{-(\alpha+s)t}dt = \left.\frac{e^{-(\alpha+s)t}}{-(\alpha+s)}\right|_{t=0^-}^{+\infty} = \frac{1}{s+\alpha}$, $\mathrm{Re}(s) > -\mathrm{Re}(\alpha)$.

$e^{-\alpha t}u(-t) \leftrightarrow \int_{-\infty}^{0^+} e^{-(\alpha+s)t}dt = \left.\frac{e^{-(\alpha+s)t}}{-(s+\alpha)}\right|_{t=-\infty}^{0^+} = \frac{-1}{s+\alpha}$, $\mathrm{Re}(s) < -\mathrm{Re}(\alpha)$.

$\frac{d}{ds}X(s) = \frac{d}{ds}\int_{\mathbb{R}} x(t)e^{-st}dt \leftrightarrow -tx(t)$.

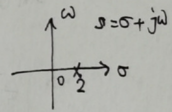


$\therefore\ t(e^{-2t}u(t) - e^{2t}u(-t)) \leftrightarrow \frac{d}{ds}\left[\frac{1}{s+2} + \frac{1}{s-2}\right] = -\left(\frac{1}{(s+2)^2} + \frac{1}{(s-2)^2}\right)$

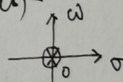
$$= \frac{-(s^2+8)}{(s+2)^2(s-2)^2}, \quad -2 < \mathrm{Re}(s) < 2.$$

(f) $e^{-\alpha|t|} = e^{-\alpha t}u(t) + e^{\alpha t}u(-t) \leftrightarrow \frac{1}{s+\alpha} - \frac{1}{s-\alpha}$, $-\mathrm{Re}(\alpha) < \mathrm{Re}(s) < \mathrm{Re}(\alpha)$.

$x(t) = -te^{2t}u(-t) \leftrightarrow \frac{d}{ds}\left[\frac{-1}{s-2}\right] = \frac{1}{(s-2)^2}$, $\mathrm{Re}(s) < 2$.

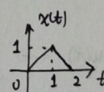


(g) $X(s) = \int_{\mathbb{R}} x(t)e^{-st}dt = \int_0^1 e^{-st}dt = \left.\frac{e^{-st}}{-s}\right|_{t=0}^{1} = \frac{e^{-s}-1}{-s}$, ~~Re(s)~~ $s \in \mathbb{C}$.

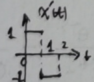


s=0：一重极点，零点

(h) $X(s) = \int_0^1 te^{-st}dt + \int_1^2 (2-t)e^{-st}dt$. $x(t) =$



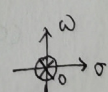
$\Rightarrow x'(t) =$



, $x(t) = \int_{-\infty}^{t} x'(t)\,dt$. 记 $x'(t) \leftrightarrow X_1(s)$.

$\Rightarrow X(s) = \int_{\mathbb{R}} x(t)e^{-st}dt = \int_{\mathbb{R}} e^{-st}dt \int_{\mathbb{R}} x'(\tau)\overset{u(t-\tau)}{\vee}d\tau = \int_{\mathbb{R}} x'(\tau)\overset{e^{-s\tau}}{\vee}d\tau \int_{\mathbb{R}} u(t-\tau)e^{-s(t-\tau)}d(t-\tau)$

$\left[u(t) \leftrightarrow \int_{0^-}^{+\infty} e^{-st}dt = \left.\frac{e^{-st}}{-s}\right|_{0^-}^{+\infty} = \frac{1}{s}, \mathrm{Re}(s) > 0\right]$ $= \frac{1}{s}X_1(s)$.



$X_1(s) = \int_0^1 e^{-st}dt - \int_1^2 e^{-st}dt = \left.\frac{e^{-st}}{-s}\right|_0^1 - \Big|_1^2 = \frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}$.

or $x'(t) = u(t) - 2u(t-1) + u(t-2) \leftrightarrow \frac{1}{s}\left[1 - 2e^{-s} + e^{-2s}\right] = \frac{(e^{-s}-1)^2}{s}$, ~~Re(s)~~ $s \in \mathbb{C}$

s=0：一重极点，二重零点

(i) $u(t) \leftrightarrow \int_{0^-}^{+\infty} e^{-st} dt = \frac{e^{-st}}{-s}\Big|_{t=0^-}^{+\infty} = \frac{1}{s}$, $\mathrm{Re}(s)>0$.

$\delta(t) = \frac{d}{dt}u(t) \leftrightarrow \int_{\mathbb{R}} \delta(t) e^{-st} dt = 1 = s \cdot \frac{1}{s}$.

$s=\sigma+j\omega$

$x'(t) \leftrightarrow \int_{\mathbb{R}} x'(t) e^{-st} dt = x(t)e^{-st}\Big|_{t=-\infty}^{+\infty} + s\int_{\mathbb{R}} x(t) e^{-st} dt = sX(s)$,

$s\in ROC\{x'(t)\}$.

$\Rightarrow x(t) = u(t)+\delta(t) \leftrightarrow \frac{1}{s}+1$, $\mathrm{Re}(s)>0$.

(j) $x(at) \leftrightarrow \int_{\mathbb{R}} x(at) e^{-st} dt \overset{\tau=at}{=\!=} \int_{\mathbb{R}} x(\tau) e^{-\frac{s}{a}\tau} \frac{d\tau}{|a|} = \frac{1}{|a|}X(\frac{s}{a})$.

$\Rightarrow \delta(3t) \leftrightarrow \frac{1}{3}$, $u(3t) \leftrightarrow \frac{1}{3}\frac{1}{s/3} = \frac{1}{s}$, $\Rightarrow x(t) \leftrightarrow \frac{1}{3}+\frac{1}{s}$, $\mathrm{Re}(s)>0$.

2. 解: $e^{s_0 t}x(t) \leftrightarrow \int_{\mathbb{R}} x(t) e^{-(s-s_0)t} dt = X(s-s_0)$, $s\in s_0+R$.

$\sin(\omega_0 t)u(t) = \frac{1}{2j}[e^{+j\omega_0 t} - e^{-j\omega_0 t}] \leftrightarrow \frac{1}{2j}\left[\frac{1}{s-j\omega_0} - \frac{1}{s+j\omega_0}\right] = \frac{\omega_0}{s^2+\omega_0^2}$, $\mathrm{Re}(s)>0$.

$\Rightarrow e^{-\alpha t}\sin(\omega_0 t)u(t) \leftrightarrow \frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$, $\mathrm{Re}(s) > -\mathrm{Re}(\alpha)$.

$\frac{d}{ds}X(s) = \int_{\mathbb{R}} -t\, x(t) e^{-st} dt \leftrightarrow -t x(t)$

(a) $e^{-t}\sin(2t)u(t) \leftrightarrow \frac{2}{(s+1)^2+4}$, $\mathrm{Re}(s)>-1$.

(b) $e t e^{-(t-1)}u(t-1) \leftrightarrow -e\frac{d}{ds}\left[\frac{e^{-s}}{s+1}\right] = -e\left[\frac{-e^{-s}}{s+1} - \frac{e^{-s}}{(s+1)^2}\right]$, $\mathrm{Re}(s)>-1$.

$[e^{-\alpha t}u(t) \leftrightarrow \frac{1}{s+\alpha}$, $x(t-t_0) \leftrightarrow \int_{\mathbb{R}} x(t-t_0)e^{-s(t-t_0)} d(t-t_0) e^{-st_0} = e^{-st_0}X(s)]$

(c) $t[u(t-1)-u(t-2)] \leftrightarrow -\frac{d}{ds}\int_1^2 e^{-st}dt = -\frac{d}{ds}\left[\frac{e^{-st}}{-s}\Big|_{t=1}^{2}\right]$

$= +\frac{d}{ds}\left[\frac{e^{-2s}-e^{-s}}{+s}\right] = \frac{-2e^{-2s}+e^{-s}}{s} - \frac{e^{-2s}-e^{-s}}{s^2}$, ~~Re(s)~~ $s\in\mathbb{C}$.

$= s^{-2}[-(1+2s)e^{-2s} + (1+s)e^{-s}]$, $s=0$: 二重极, 7/2重零点.

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2. 解：(d) $\sin[2(t-1)+1]\,u(t-1) = \sin(2(t-1))\,u(t-1)\cos 1 + \cos(2(t-1))\,u(t-1)\sin 1$.

$\cos(\omega_0 t)u(t) \leftrightarrow \frac{1}{2}[e^{j\omega_0 t}+e^{-j\omega_0 t}] = \frac{1}{2}\left[\frac{1}{s-j\omega_0}+\frac{1}{s+j\omega_0}\right] = \frac{s}{s^2+\omega_0^2}$, $Re(s)>0$.

$\therefore\ x(t) \leftrightarrow \frac{2e^{-s}\cos 1}{s^2+4} + \frac{se^{-s}\cdot\sin 1}{s^2+4}$, $Re(s)>0$.

3. 解：(a) $\frac{\frac{1}{3}\cdot 3}{s^2+3^2}$, $Re(s)>0 \leftrightarrow \frac{1}{3}\sin(3t)u(t)$.

(b) $\cos(\omega_0 t)u(-t) = \frac{1}{2}[e^{j\omega_0 t}+e^{-j\omega_0 t}]u(-t) \leftrightarrow \frac{1}{2}\left[\frac{-1}{s-j\omega_0}+\frac{-1}{s+j\omega_0}\right]$

$[e^{-\alpha t}u(-t) \leftrightarrow \int_{-\infty}^{0^+} e^{-(\alpha+s)t}dt = \frac{e^{-(\alpha+s)t}}{-(\alpha+s)}\Big|_{-\infty}^{0^+} = \frac{1}{s+\alpha}] = \frac{-s}{s^2+\omega_0^2}$, $Re(s)<0$

$Re(s)<-Re(\alpha)$.

$\Rightarrow \frac{s}{s^2+9}$, $Re(s)<0 \leftrightarrow -\cos(3t)u(-t)$.

(c) $\frac{s+1}{(s+1)^2+3^2} \overset{Re(s)<-1}{\longleftrightarrow} \cancel{e^{-s}} - e^{-t}\cos(3t)u(-t)$.

(d) $\frac{-1}{s+3}+\frac{2}{s+4}$, $-4<Re(s)<-3 \leftrightarrow e^{-3t}u(-t)+2e^{-4t}u(t)$.

(e) $\frac{-1}{s+2}+\frac{2}{s+3}$, $-3<Re(s)<-2 \leftrightarrow e^{-2t}u(-t)+2e^{-3t}u(t)$

(f) $\frac{3(s-\frac{1}{2})+2\cdot(\frac{\sqrt{3}}{2})^2}{(s-\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}+1$, $Re(s)>\frac{1}{2}$

$$\begin{array}{r|l} & \quad 1 \\ s^2-s+1 & s^2+2s+1 \\ & s^2-s+1 \\ \hline & \quad 3s \end{array}$$

$\leftrightarrow 3e^{\frac{1}{2}t}\cos(\frac{\sqrt{3}}{2}t)u(t) + 2e^{\frac{1}{2}t}\sin(\frac{\sqrt{3}}{2}t)u(t) + \delta(t)$.

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3. 解：(g) $\frac{(s+1)^2-3s}{(s+1)^2} = 1 + \frac{A}{s+1} + \frac{B}{(s+1)^2}$，$B = -3s\Big|_{s=-1} = 3$，

$A = \frac{d}{ds}\left[-3s\right]\Big|_{s=-1} = -3 \Rightarrow X(s) = 1 - \frac{3}{s+1} + \frac{3}{(s+1)^2}$，$\text{Re}(s) > -1$

$u(t) \leftrightarrow \frac{1}{s}$，$\text{Re}(s)>0$.　　$\leftrightarrow \delta(t) - \left[3e^{-t} + 3te^{-t}\right]u(t)$.

$t^n u(t) \leftrightarrow (-1)^n \frac{d^n}{ds^n}(s^{-1}) = (-1)^n \cdot (-1)^n\, n!\, s^{-(n+1)} = \frac{n!}{s^{n+1}}$，$\text{Re}(s) > 0$.

4. 解：(a) $\frac{6}{s+4} + \frac{-3}{s+2}$，$\text{Re}(s) > -2 \leftrightarrow (6e^{-4t} - 3e^{-2t})u(t)$.

(b) $\frac{s+3}{(s+1)^3(s+2)} = \frac{A_0}{(s+1)^3} + \frac{A_1}{(s+1)^2} + \frac{A_2}{s+1} + \frac{B}{s+2}$，$\text{Re}(s) > -1$

$A_0 = \frac{s+3}{s+2}\Big|_{s=-1} = 2$，$A_1 = \frac{d}{ds}\left[1 + \frac{1}{s+2}\right]\Big|_{s=-1} = -1$，

$A_2 = \frac{1}{2!}\frac{d^2}{ds^2}\left[1 + \frac{1}{s+2}\right]\Big|_{s=-1} = \frac{1}{2}\cdot 2 \cdot (+1) = +1$，

$B = \frac{s+3}{(s+1)^3}\Big|_{s=-2} = -1$. $\therefore X(s) \leftrightarrow e^{-t}u(t)\left[t^2 - t + 1\right] - e^{-2t}u(t)$.

(c) $\frac{1}{4s(s^2+1)} = \frac{1/4}{s} + \frac{As+B}{s^2+1} + \frac{0}{s^2+1} = \frac{(\frac{1}{4}+A)s^2 + Bs + \frac{1}{4}}{s(s^2+1)}$

$\Rightarrow A = -1/4,\ B = 0 \Rightarrow \frac{1}{4s(s^2+1)} = \frac{1/4}{s} - \frac{\frac{1}{4}s}{s^2+1}$，$\text{Re}(s) > 0$

$\leftrightarrow \left(\frac{1}{4}e^{-0t} - \frac{1}{4}\cos t\right)u(t)$

$\Rightarrow \frac{e^{-s}}{4s(s^2+1)}$，$\text{Re}(s)>0 \leftrightarrow \frac{1}{4}u(t-1)\left(1 - \cos(t-1)\right)$.

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5. 解: (a) $\dfrac{1}{(s^2+3)^2} = \dfrac{-1}{2s}\dfrac{d}{ds}\left[\dfrac{1}{s^2+3}\right]$, $Re(s)>0$

$\dfrac{d}{ds}X(s) = \int_{\mathbb{R}} -t\,x(t)e^{-st}dt \leftrightarrow -tx(t)$, $\sin(\omega_0 t)u(t) \leftrightarrow \dfrac{\omega_0}{s^2+\omega_0^2}$, $Re(s)>0$.

$\int_{-\infty}^{t} x(t)dt = x(t) * u(t) \leftrightarrow \dfrac{1}{s}X(s)$,

$f(t)*g(t) \leftrightarrow \int_{\mathbb{R}} e^{-st}dt \int_{\mathbb{R}} f(\tau)g(t-\tau)d\tau$

$= \int_{\mathbb{R}} f(\tau)e^{-s\tau}d\tau \int_{\mathbb{R}} g(t-\tau)\, e^{-s(t-\tau)} d(t-\tau) = F(s)G(s)$.

$\Rightarrow -\dfrac{1}{2}\dfrac{1}{s}\dfrac{d}{ds}\left[\dfrac{\frac{1}{\sqrt{3}}\sqrt{3}}{s^2+3}\right] \leftrightarrow -\dfrac{1}{2}\int_{-\infty}^{t} -t\dfrac{1}{\sqrt{3}}\sin(\sqrt{3}t)\,u(t)\,dt$

$= +\dfrac{u(t)}{2\sqrt{3}}\int_0^t t\sin(\sqrt{3}t)dt = \dfrac{u(t)}{2\sqrt{3}}\left[\begin{array}{c|c|c} t & 1 & 0 \\ \sin(\sqrt{3}t) \oplus & -\frac{1}{\sqrt{3}}\cos(\sqrt{3}t) \ominus & -\frac{1}{3}\sin(\sqrt{3}t) \oplus \end{array}\right]_{t=0}^{t}$

$= \dfrac{u(t)}{2\sqrt{3}}\left[-\dfrac{t}{\sqrt{3}}\cos(\sqrt{3}t) - \dfrac{1}{3}\sin(\sqrt{3}t)\right]$.

6. 解: $\int_{0^-}^{+\infty} x'(t)e^{-st}dt = x(t)e^{-st}\Big|_{t=0^-}^{+\infty} + s\int_{0^-}^{+\infty} x(t)e^{-st}dt \overset{Re(s)>0}{=\!=} sX(s) - x(0^-)$

$\Rightarrow \dfrac{d^n}{dt^n}x(t) \leftrightarrow s^n X(s) - \sum_{k=0}^{n-1} s^{n-1-k} x^{(k)}(0^-)$.

$\therefore (s^2+3s+2)Y(s) - (sy(0^-) + y'(0^-) + 3y(0^-)) = (s+4)X(s)$.

(a) $x(t) = u(t) \leftrightarrow X(s) = \dfrac{1}{s} \Rightarrow Y_{zs}(s) = \dfrac{(s+4)}{s(s+1)(s+2)} = \dfrac{2}{s} + \dfrac{-3}{s+1} + \dfrac{1}{s+2} \longleftrightarrow (2-3e^{-t}+2e^{-2t})u(t) =: y_{zs}(t)$,

$Y_{zi}(s) = \dfrac{1}{(s+1)(s+2)} = \dfrac{1}{s+1} + \dfrac{-1}{s+2} \leftrightarrow (e^{-t}-e^{-2t})u(t) =: y_{zi}(t)$.

(b) $Y_{zi}(s) = \dfrac{s+4}{(s+1)(s+2)} = \dfrac{3}{s+1} + \dfrac{-2}{s+2} \leftrightarrow y_{zi}(t) = (3e^{-t}-2e^{-2t})u(t)$,

$X(s) = \dfrac{1}{s+2} \Rightarrow Y_{zs}(s) = \dfrac{s+4}{(s+1)(s+2)^2} = \dfrac{3}{s+1} + \dfrac{-2}{(s+2)^2} + \dfrac{-3}{s+2} \leftrightarrow y_{zs}(t) = u(t)\left[3e^{-t} - (2t+3)e^{-2t}\right]$.

－6－

2 Chapter 9-2

ICE2301 Homework (Chapter 9-2) 2022.05.19 (due)

7. 解: (a)(b) $H(s) = \frac{s-1}{s+1} = 1 - \frac{2}{s+1}$, $\mathrm{Re}(s) > -1$. (casual),

$y(t) = e^{-2t}u(t) \leftrightarrow \frac{1}{s+2}$, $\mathrm{Re}(s) > -2$. $=: Y(s)$,

$Y(s) = X(s)H(s)$, $\boxed{\mathrm{ROC}\{X(s)\} \cap \mathrm{ROC}\{H(s)\} \neq \phi.}$

$\Rightarrow X(s) = Y(s)/H(s) = \frac{s+1}{(s+2)(s-1)}$, ROC: (i) $\mathrm{Re}(s) > 1$, 或 (ii) $-2 < \mathrm{Re}(s) < 1$.

(i) $\mathrm{ROC}\{X(s)\} = \{\mathrm{Re}(s) > 1\}$, (不含虚轴).

~~此时 x(t) =~~ 此时 $X(s) = \frac{1/3}{s+2} + \frac{2/3}{s-1} \leftrightarrow \frac{1}{3}e^{-2t}u(t) + \frac{2}{3}e^{t}u(t)$,

不满足 $\int_{\mathbb{R}} |x(t)|\,dt < +\infty$, 但确有 $y(t) = x(t) * h(t) = e^{-2t}u(t)$.

(ii) $\mathrm{ROC}\{X(s)\} = \{-2 < \mathrm{Re}(s) < 1\}$, (含虚轴)

此时 $X(s) = \frac{1/3}{s+2} + \frac{2/3}{s-1} \leftrightarrow -\frac{2}{3}e^{t}u(-t) + \frac{1}{3}e^{-2t}u(t)$,

满足 $\int_{\mathbb{R}} |x(t)|\,dt < +\infty$, 且确有 $y(t) = x(t) * h(t) = e^{-2t}u(t)$.

☆ (iii) $\mathrm{ROC}\{X(s)\} = \{\mathrm{Re}(s) < -2\}$,

此时 $X(s)$ 与 $H(s)$ 无公共收敛域, $x(t) = \left(-\frac{2}{3}e^{t} - \frac{1}{3}e^{-2t}\right)u(-t)$,

$h(t) = \delta(t) - 2e^{-t}u(t)$, $y(t) = x(t) * h(t) \to \infty$! 非法信号!

∴ $x(t)$ 可为 (i)(ii), 不能为 (iii)

☆ **Remark** $x_1(t) * x_2(t) \leftrightarrow X_1(s)X_2(s)$, ROC 至少包含 $R_1 \cap R_2$.

若 $X_1(s), X_2(s)$ 发生零极点抵消, 则收敛域可能扩大.(例如, 本题 (i)(ii))

若 $X_1(s)X_2(s)$, $R_1 \cap R_2 = \emptyset$, 则上述卷积性质无法正常使用,

可能不成立.(e.g. 本题 (iii))

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7.(c) 解：已知有一稳定 LTI 系统 $H_2(s) \leftrightarrow h_2(t)$ 满足

$$y(t) = e^{-2t}u(t) \longrightarrow \boxed{H_2(s)} \longrightarrow x(t) \leftrightarrow X(s) = \frac{s+1}{(s+2)(s-1)},\quad ROC = R_2 \text{ 待定.}$$

⇒ $\cancel{H_2(s)=}$ $Y(s)H_2(s) = X(s)$，且 $X(s)$ 的 $ROC = R_2$ 至少包含 ROC: $Y(s) \cap H_2(s)$.

⇒ $H_2(s) = X(s)/Y(s) = \dfrac{s+1}{s-1}$，ROC: $Re(s) < 1$ [∵ H_2 稳定 ⇒ ROC含虚轴]

⇒ $h_2(t) = \delta(t) - 2e^{t}u(-t)$，且 $X(s)$ 的 ROC 至少包含 $-2 < Re(s) < 1$.

⇒ $X(s) = \dfrac{s+1}{(s+2)(s-1)} = \dfrac{-1/3}{s+2} + \dfrac{2/3}{s-1}$，$-2 < Re(s) < 1$

$\longleftrightarrow$ $x(t) = -\frac{1}{3}e^{-2t}u(t) - \frac{2}{3}e^{t}u(-t)$.

8. 解：$x(t) = e^{-|t|} = e^{t}u(-t) + e^{-t}u(t) \leftrightarrow \dfrac{-1}{s-1} + \dfrac{1}{s+1} = \dfrac{-2}{(s+1)(s-1)} =: X(s)$，$-1 < Re(s) < 1$.

⇒ $Y(s) = X(s)H(s) = \dfrac{-2}{(s^2+2s+2)(s-1)} = \dfrac{As+B}{s^2+2s+2} + \dfrac{-2/5}{s-1}$

⇒ $(A - \frac{2}{5})s^2 + (-\frac{4}{5} - A + B)s + (-\frac{4}{5} - B) = -2$

⇒ $A = 2/5$，$B = 6/5$

⇒ $Y(s) = \dfrac{\frac{2}{5}s + \frac{6}{5}}{s^2+2s+2} - \dfrac{\frac{2}{5}}{s-1}$，$-1 < Re(s) < 1$.

[$Roc\{H(s)\} = \{Re(s) > -1\}$，$Roc\{X(s)\} = \{-1 < Re(s) < 1\}$，

⇒ $Roc\{Y(s)\}$ 至少包含 $-1 < Re(s) < 1$]

⇒ $Y(s) = \dfrac{\frac{2}{5}(s+1)}{(s+1)^2+1} + \dfrac{\frac{4}{5}\cdot 1}{(s+1)^2+1^2} - \dfrac{2/5}{s-1} \leftrightarrow e^{-t}u(t)\left[\frac{2}{5}\cos t + \frac{4}{5}\sin t\right] + \frac{2}{5}e^{t}u(-t) =: y(t)$.

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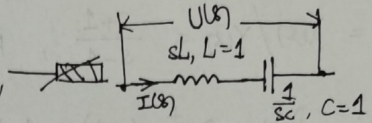
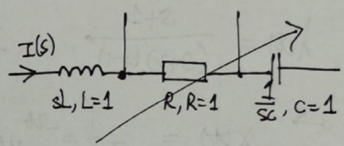
9. 解：(a) $H(s)=\dfrac{U_2(s)}{U_1(s)}=\dfrac{\frac{R}{sC}\Big/\left(\frac{1}{sC}+R\right)}{R+\frac{1}{sC}+\frac{R}{sC}\Big/\left(\frac{1}{sC}+R\right)}$,

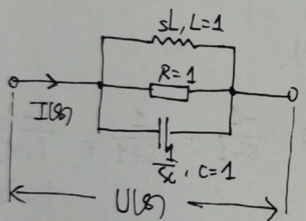
(b) $H(s)=\dfrac{U_2(s)}{U_1(s)}=\dfrac{\frac{1}{sC}-R}{\frac{1}{sC}+R}$.

10. 解：(a) $H(s)=\dfrac{U(s)}{I(s)}=s+\frac{1}{s}$,



(b) $H(s)=\dfrac{U(s)}{I(s)}=\dfrac{1}{s+1+1/s}$,





11. 解：$\delta(t)\leftrightarrow X_1(s)=1\xrightarrow{LTI}Y_1(s)=X_1(s)H_1(s)+Y_{zi}(s)=1+\frac{1}{s-1}$,

$u(t)\leftrightarrow X_2(s)=\frac{1}{s}\xrightarrow{LTI}Y_2(s)=\underbrace{X_2(s)H(s)}_{[\text{ROC 至少与}\{Re(s)>1\}\text{交叠}]}+Y_{zi}(s)=\frac{3}{s-1}$, $Re(s)>1$.

$\Rightarrow\ \left(1-\frac{1}{s}\right)H(s)=1-\frac{2}{s-1}$

$\Rightarrow\ H(s)=\frac{s-3}{s-1}\Big/\frac{s-1}{s}=\frac{s-3}{s}=1-\frac{3}{s}$, $Re(s)>0$.

~~(a)~~ $\Rightarrow\ Y_{zi}(s)=Y_1(s)-H(s)=\frac{1}{s-1}+\frac{3}{s}$, $Re(s)>\cancel{0}\ 1$

(a) $Y(s)=H(s)X(s)+Y_{zi}(s)=\frac{s-3}{s}\frac{1}{s+2}+\left(\frac{1}{s-1}+\frac{3}{s}\right)$, $Re(s)>1$

$=\frac{-3/2}{s}+\frac{5/2}{s+2}+\frac{1}{s-1}+\frac{3}{s}$

$\leftrightarrow y(t)=\left(\frac{3}{2}+\frac{5}{2}e^{-2t}+e^{t}\right)u(t)$.

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11(b) 解：$Y(s) = H(s)X(s) + Y_{zi}(s) = \underbrace{\frac{s-3}{s}\,\frac{2}{s}}_{=\frac{2}{s}-\frac{6}{s^2}}e^{-s} + \frac{1}{s-1} + \frac{3}{s}$, $\mathrm{Re}(s) > 1$

$\Rightarrow y(t) = (2 - 6(t-1))u(t-1) + (e^{t} + 3)u(t).$

12. 解：$Y(s) = \frac{2}{3}\frac{1}{s-2} + \frac{1}{3}\frac{1}{s+1}$, $-1 < \mathrm{Re}(s) < 2$.

$X(s) = \frac{s+2}{s-2} = 1 + \frac{4}{s-2}$, $\mathrm{Re}(s) < 2 \longleftrightarrow x(t) = \delta(t) - 4e^{2t}u(-t)$. [$x(t)=0, t>0$]

(a) $X(s)H(s) = Y(s)$，$X(s)$与$H(s)$的ROC的交非空.

$\Rightarrow H(s) = Y(s)/X(s) = \frac{s-\cancel{2}3}{(s+1)(s-2)} \Big/ \frac{s+2}{s-2} = \cancel{s + \frac{5}{3} - \frac{1}{s-1}}$, $s+1-\frac{2}{s+1}$, $\mathrm{Re}(s) > -1$.

[若 $H(s)$ 的 ROC 取 $\{\mathrm{Re}(s) < -1\}$；则 $Y(s)$ 的 ROC 成为 $\{\mathrm{Re}(s) < -1\}$，与已知条件不符]

(b) $H(s) = s+1-\frac{2}{s+1} \overset{\mathrm{Re}(s)>-1}{\longleftrightarrow} \delta'(t) + \delta(t) - 2e^{-t}u(t) =: h(t)$.

(c) $e^{st} \rightarrow \boxed{H(s)} \longrightarrow y_{zs}(t) = e^{st} * h(t) = \int_{\mathbb{R}} e^{s(t-\tau)}h(\tau)\,d\tau = H(s)e^{st}$.

$\therefore x(t) = e^{3t} \rightarrow \boxed{H(s)} \rightarrow y_{zs}(t) = H(s)\Big|_{s=3} e^{st} = \frac{7}{2}e^{3t}$.

13. 解：(a) $Y(s) = H(s)X(s) = \frac{10\cancel{s}}{(s+1)(s+2)\cancel{s}}$, $\mathrm{Re}(s) > -1$ [注：$H(s)$ 必是因果的，不然 ROC 和 $X(s)$ 的交为空，$y(t)$ bloom!]

$\longleftrightarrow y(t) = 10(e^{-t} - e^{-2t})u(t) =$ natural response. forced response $= 0$. [$X(s)$的极点已被$H(s)$的零点抵消]

(b) $Y(s) = H(s)X(s) = \cancel{\frac{10s}{(s+1)^2(s+2)}}\ \frac{s}{(s+1)(s+2)}\,\frac{10}{s^2+1}$, $\mathrm{Re}(s) > 0$

$= \frac{-5}{s+1} + \frac{4}{s+2} + \frac{s+3}{s^2+1}$, $\mathrm{Re}(s) > 0$

$\longleftrightarrow y(t) = (\underbrace{-5e^{-t} + 4e^{-2t}}_{\text{natural response}} + \underbrace{\cos t + 3\sin t}_{\text{forced response.}})u(t).$

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☆ 13. 另解：设 $H(s)=\dfrac{\prod_j (s-z_j)}{\prod_i (s-p_i)}$（是有理真分式）是因果稳定 LTI 系统的 system function

$\Rightarrow \mathrm{Re}(p_i)<0,\ \forall i$. 在输入 $x(t)=E\sin(\omega_0 t)u(t) \leftrightarrow \dfrac{E\omega_0}{s^2+\omega_0^2},\ \mathrm{Re}(s)>0 =: X(s)$

下的零状态响应 $$Y(s)=H(s)X(s)=\underbrace{\frac{K_1}{s+j\omega_0}+\frac{K_2}{s-j\omega_0}}_{\text{强迫响应}}+\underbrace{\sum_i \frac{\tilde{K}_i}{s-p_i}+\dots}_{\text{自由响应}}$$

$$\Rightarrow K_{1,2}=(s\pm j\omega_0)H(s)X(s)\Big|_{s=\mp j\omega_0}=H(\mp j\omega_0)\frac{E\omega_0}{\mp 2j\omega_0}.$$

因 $\mathrm{Re}(p_i)<0$，故自由响应是暂态响应.

$$\Rightarrow \text{稳态响应}=\lim_{t\to+\infty}y(t)=\text{强迫响应}=\left[H(-j\omega_0)\frac{E}{-2j}e^{-j\omega_0 t}+H(j\omega_0)\frac{E}{2j}e^{j\omega_0 t}\right]u(t).$$

因 $h(t)$ 是实的 $\Leftrightarrow h(t)=h^*(t) \Leftrightarrow H(s)=H^*(s^*) \Rightarrow H(j\omega_0)=H^*(j\omega_0)$.

$\therefore H(-j\omega_0)=|H(j\omega_0)|e^{-j\measuredangle H(j\omega_0)}$.

$\Rightarrow$ 强迫（稳态）响应 $=|H(j\omega_0)|\,E\sin(\omega_0 t+\measuredangle H(j\omega_0))$.

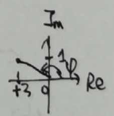
对于余弦输入 $x(t)=E\cos(\omega_0 t)$，有类似结论.

故称 $H(j\omega_0)=H(s)\big|_{s=j\omega_0}$ 为因果稳定 LTI 系统 $H(s)$ 的频率响应.

(a)

$x(t)=10\cos(0\cdot t)u(t),\ H(s)\big|_{s=j0}=0 \Rightarrow$ 强迫响应 $=0$.

(b) $x(t)=10\sin(1\cdot t)u(t),\ H(s)\big|_{s=j1}=\dfrac{j}{1+j3}=\dfrac{+3+j}{10}$



$\Rightarrow H(s)\big|_{s=j1}=H_1e^{j\varphi_0},\ H_1=\dfrac{1}{\sqrt{10}},\ \cos\varphi_0=+3/\sqrt{10},\ \sin\varphi_0=1/\sqrt{10}$

$\Rightarrow$ 稳态响应 $=10\cdot\dfrac{1}{\sqrt{10}}\sin(t+\varphi_0)u(t)=(+3\sin t+\cos t)u(t)$.

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14. 解： $H(s) = \dfrac{A(s-0)}{(s+1)^2 + \frac{3}{4}}$ ， $Re(s) > -1$.

$2 = h(0^+) = \lim\limits_{s\to\infty} sH(s) = A \Rightarrow H(s) = \dfrac{2s}{(s+1)^2 + 3/4}$ ， $Re(s) > -1$. [因果，稳定]

$\Rightarrow H(s)\Big|_{s=j\frac{\sqrt{3}}{2}} = \dfrac{\sqrt{3}}{1+j\sqrt{3}} = \dfrac{\sqrt{3}}{2}e^{j\frac{\pi}{6}}$.

$\Rightarrow$ 稳态（强迫）响应 $= \dfrac{\sqrt{3}}{2}\sin\left(\dfrac{\sqrt{3}}{2}t + \dfrac{\pi}{6}\right)u(t)$. （p.11）

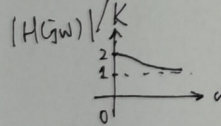
15. 解： $H(s) = K\dfrac{\prod_i (s - z_i)}{\prod_j (s - p_j)} = K\dfrac{\left(\prod_i N_i\right)e^{j\sum_i \varphi_i}}{\left(\prod_j M_j\right)e^{j\sum_j \psi_j}}$,

$s - z_i = N_i e^{j\varphi_i}$, $s - p_j = M_j e^{j\psi_j}$.

$\Rightarrow |H(s)| = K\dfrac{\prod_i N_i}{\prod_j M_j}$, $\measuredangle H(s) = \sum_i \varphi_i - \sum_j \psi_j$.

(a) $s = j\omega$. $\omega = 0$: $\frac{1}{K}|H(j\omega)| = 2$, $\measuredangle H(j\omega) = 0$

$+\infty$: 1, 0.



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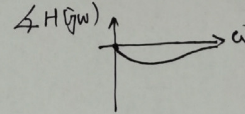

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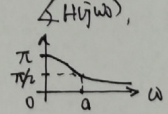
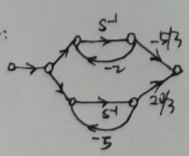
(b) $s = j\omega$. $\omega = \cancel{0}$: $\frac{1}{K}|H(j\omega)| = 1$,



16. $H(s) = \dfrac{5(s+1)}{s^2+7s+10} = \underbrace{\dfrac{-5/3}{s+2} + \dfrac{20/3}{s+5}}_{\text{并联}} = \underbrace{\dfrac{5}{s+2}\cdot\left(1 + \dfrac{-4}{s+5}\right)}_{\text{级联}} = \underbrace{\dfrac{5s^{-1} + 5s^{-2}}{1 - (7s^{-1} + 10s^{-2})}}_{\text{直接}}$

并：



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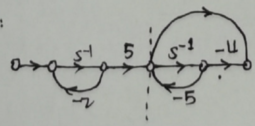

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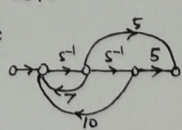


17. 解：$H = \frac{1}{\Delta}\sum_i g_i \Delta_i$， $\Delta = 1 - \sum L_a + \sum L_b L_c - \cdots$

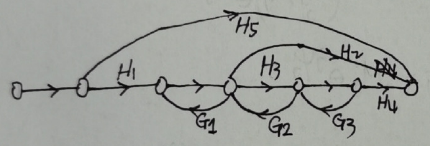
令 $\Delta = 1 - (G_1 + G_2 H_3 + G_3) + G_1 G_3$， ⇒ 三个环路，且 G_1-环和 G_3-环不接触.

令 $g_1 \Delta_1 := H_5[\cdots] = H_5 \Delta$ ⇒ 前向通路1：增益 H_5，与所有环路接触.

$g_2 \Delta_2 := H_1 H_2 (1 - G_3)$ ⇒ 前向通路2：增益 $H_1 H_2$，与 G_3-环不接触，
与 G_1，G_2-环接触.

$g_3 \Delta_3 := H_1 H_3 H_4$ ⇒ 前向通路3：增益 $H_1 H_3 H_4$，与所有环路接触.

经尝试，得一种可能的结构图：



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References